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AN EQUATION FOR ONSET OF SNOW IN A CLOUD

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ABSTRACT

Homogeneous nucleation theory is employed in Fletcher (1970) to arrive at a formula for formation of ice from water. We identify T with the "snow point" and derive the equation in a few short steps. The formula is (5.16) from Fletcher solved for T.

$$T = (D\ \Delta Sv\ \Delta T) / (k\ v\ n_s\ a)$$

Then, using some other information, the result is.

$$T = D\ (7.15 \times 10^7)/(k\ a\ n_s\ \Delta T^{0.69})$$

It should be noted that $\Delta T = To - T$, is the underooling for formation of ice in the cloud.

KEYWORDS
"onset of snow" "homogeneous nucleation theory" "ice formation" supercooling "melting temperature of water" "growth velocity of ice-water interface"

I. INTRODUCTION

From thermodynamic considerations, we have (4.29) as (1) from Fletcher.

$$\Delta G_V = - \langle \Delta Sv \rangle\ \Delta T \qquad (1)$$

Also, Fletcher maintains that

$$\langle \Delta Sv \rangle \approx (1.13 - 0.004\ \Delta T) \times 10^7 \text{ erg/cc-deg} \qquad (2)$$

where $\Delta T = To - T$ is the underooling, typically less than 0.5 ˚C, so $0.004\ \Delta T$ is neglected, Hillig (1958), p. 351. To is the melting temperture of ice and the pressure dependence of To is well-known.

RESULTS

According to Fletcher, p. 109, there obtains this formula for the velocity, v, of ice crystal growth, which the author posits to be the same for a cloud of supercooled water.

$$v = (D/kT)\ (\mu_I - \mu_S)\ (n'/n_S\ l) \qquad (3)$$

Then, on page 105, Fletcher has.

$$\Delta G_V = - n_S\ (\mu_I - \mu_S) \qquad (4)$$

Putting (3) and (4) together obtains (with l approximately equal to a).

$$v = - (D/kT)\ \Delta G_V\ (1/a)\ \ (n'/n_S^2) \qquad (5)$$

Then, Fletcher notes that $n' \approx n_S$ and we use (1).

We arrive at the following equation, solving for T.

$$T = (D\ \Delta S_V\ \Delta T)/(k\ v\ n_S\ a) \qquad (6)$$

From Hillig and Turnbull (1956) there is this formula for the velocity of ice formation.

$$\mathbf{v = (0.158 \pm 0.009) \times \Delta T^{(1.69 \pm 0.03)}\ cm/sec} \qquad (7)$$

Combining (2), (6) and (7) we get the final equation for T.

$$\mathbf{T = D\ (7.15 \times 10^{7})\ /(k\ a\ ns\ \Delta T^{0.69})} \qquad (8)$$

II. DISCUSSION

The author previously derived an equation for the onset of rain in a cloud, identifying it with the dew point, where water droplets form, Jennings (2021).

$$\mathbf{T - To = (3\ k\ To^{2} / \sigma o\ ao)\ (P^{*}H2O\ /\ Pair)\ RH} \qquad (9)$$

The object is to solve for To, dew point, but not as a quadratic. It has to be done numerically because some variables depend on T and others on To.

This paper is concerned with formation of ice nuclei in a cloud. The mathematics for classical nucleation theory goes back to Lord Kelvin, Josiah Willard Gibbs and Max Volmer. The author's first work in nucleation was in Jennings and Middleman (1985).

Eq. (8) is presumed to be accurate for clouds assuming experimental results in a bath mimics ice formation in clouds. Obviously it takes a computer to solve for T in (8).

III. CONCLUSION

Possibly Eq. (8) can be used to know when to cloud-seed with AgI or CH3-O- Na+, dry ice, NaCl, etc. Above all, this paper is in honor of God, who generously provided much snow in the Sierra, even though PROPOSITION 1 passed by a wide margin in the November 2022 election in California.

IV. ACKNOWLEDGMENT

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NOMENCLATURE FOR EQUATIONS (6) and (8)

a atomic spacing of ice (cm)
D self-diffusion coefficient of water (m^2/sec)
k Boltzmann constant (Joules/Kelvin)
n_S number density of ice (molecules/cc)
$\langle \Delta S_V \rangle$ average entropy of fusion over the supercooling range $\Delta T = To - T$ (Joules/Kelvin)
T temperature Kelvin of the bath or supercooling limit in the cloud
To melting temperature Kelvin of ice
v growth velocity of ice-water interface (cm/sec)

ADDENDUM

This paper is speculative, so the author admits there are shortcomings. First ΔT is unknown for the cloud. Second, it is only an assumption that laboratory experiments with ice formation mimic what happens in a snow-producing cloud. Finally, the self-diffusion coefficient needs to be for water at, say, - 20 ˚C, so that might be done by extrapolation.

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