

## A MINI-REVIEW AS APPLIED TO EULER'S CONVEX POLYHEDRA FORMULA AND TWO OTHERS

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**Abstract:** We show the correspondence between Euler-Gibbs-DNA Polyhedra equations. It is possible to relate these three equations.

Leonhard Euler  $V = 2 + E - F$  (convex polyhedra)

Josiah Willard Gibbs  $F = 2 + C - P$  (phase rule)

DNA Polyhedra  $s = 2 + c - \mu$  (biology)

The sources are presented which have the proofs for the three equations and the details for the correspondence. There is a corresponding states between them as math bridges geometry, chemistry and biology.

**Key Words:** "convex polyhedra" "Gibbs phase rule" "DNA polyhedra" "corresponding states"

### I. Introduction

There is a pattern between mathematics, chemistry and biology and it follows Euler's convex polyhedra formula. Euler's formula relates  $V$  vertices,  $E$  edges and  $F$  faces of the polyhedron, the most simple being the convex one. See Fig. 1 for an equilateral prism and a cube being deformed while  $V$ ,  $E$  and  $F$  are invariant. Then, for the Gibbs phase rule in chemistry, the vertices  $V$  correspond to the degrees of freedom  $F$  because the convex vertices can have some movement in  $(x, y, z)$  space as shown in Fig. 1. Another parallel construct is that of DNA Polyhedra with  $s$  Siefert circles that are analogous to  $V$  vertices, as they appear to be able to move in  $(x, y, z)$  space.

### II. Results

The proof for Euler's theorem is in the literature [1] and the proof for the Gibbs phase rule is in [2] pages 211-216. Hu, et al [3] have the proof for the DNA Polyhedra formula.

Now, it remains to set up the correspondence as in the Abstract. Fig. 1 outlines how  $V$  in Euler corresponds with  $F$  in Gibbs. Then, think of the phases as pools bounded by components, so  $P$  corresponds with  $F$ . One can think of the faces as having edges and the phases as having components.

Thus, Euler and Gibbs are mathematically parallel. In Hu, et al [3] the correspondence is laid out in equations (4) – (11).

### III. Discussion

One thing to examine is to understand why the three equations in the Abstract correspond. It is like the principal of corresponding states [4], of which the van der Waals equation of state [5] page 168 is a prime example. Hu, et al [3] notes that, by October 2011, for DNA Polyhedra there is: cube, tetrahedron, octahedron, dodecahedron, icosahedrons and buckyball. What I have outlined in the Abstract is the simple case. Euler noted there is another formula for non-convex polyhedra, but that is out of the scope of this article.

### IV. Conclusion

The author should mention an article that is included in his efforts in physical chemistry [6], which is the theory for the Bubble Nucleation in Polymer Solutions data. The author has an interest in mathematics and is fascinated by Eulers formula applying to convex polyhedra.

### **V. Acknowledgments**

The Second Coming will save all the living on earth. Paula Campbell is a faithful friend of mine since January 1987.

### **References**

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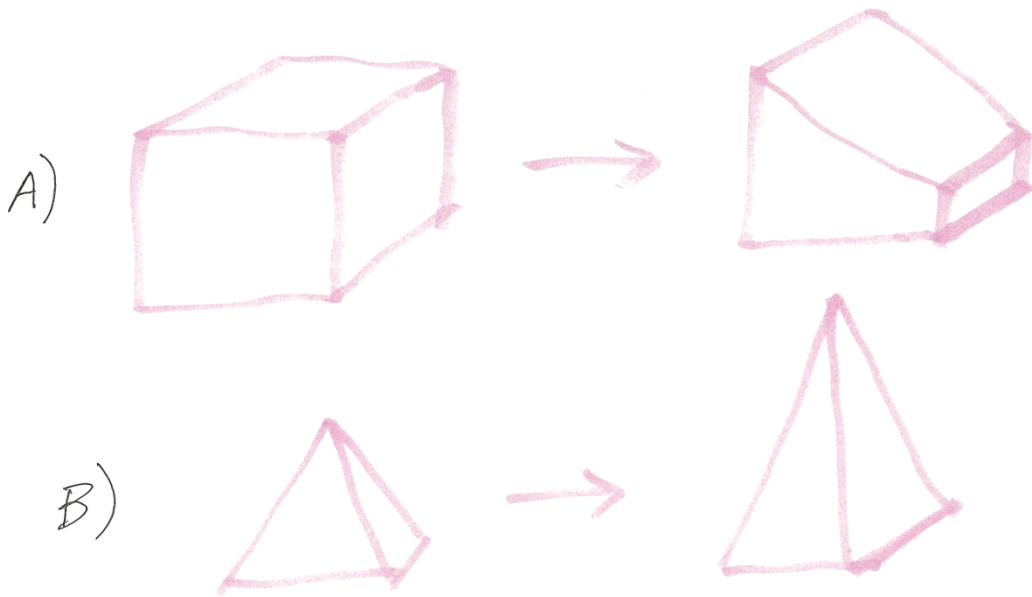


Fig. 1 A)  $V$ ,  $E$  and  $F$  are invariant  
by deformation of cube

B) same as A) for equilateral pyramid

Meaning:  $V$  (vertices) are likened to  $F$   
(degrees of freedom). Thus,  $V$  in

Euler corresponds to  $F$  in Gibbs.

Vertices have some movement in  $(x, y, z)$  space.