

The Dew Point as Nucleation Limit in a Cloud

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ABSTRACT: Consider a rain cloud near the dew point, where droplets appear from the water vapor. Then, the nucleation equation previously derived for polymer solutions could be thought of as applying in reverse, water vapor--->water droplets in a cloud. The “solution” would then be the air phase with water vapor mixed with the constituents of the atmosphere and the new phase is the pure water droplet. This paper explains how the author’s nucleation equation pertains to the rain-producing cloud in reverse in equation (5), where T is the temperature of the cloud, To, the temperature it needs to cool to for water droplets to form and n_2/n , the vapor pressure of water/air pressure. Presumably, atmospheric chemists have access to proper data from altostratus and nimbostratus clouds and could better predict when clouds will nucleate with this very simple formula.

SEARCH TERMS

“homogeneous nucleation” rain “dew point” cloud “relative humidity” “water vapor”

I. AIMS AND SCOPE

From ancient times, man has tried to understand clouds. Indeed, in PSALMS 68:34 “his strength is in the clouds.” (KJV) The scientist thinks God’s thoughts after Him and here we offer a simple formula.

By the end of Summer 2020, the author John H. Jennings had fully established his formula for bubble nucleation in polymer solutions (1) [see Ref. 1] and its precursor (2) [see Refs. 2 and 3]. We perform an imaginary experiment where the tiny H₂O droplet is formed from water vapor in air as it cools below the dew point. In classical nucleation theory, a droplet rises in temperature and explosively boils at the limit of superheat, but here there is supercooling and all of the same mathematics is presumed to apply to the process. See Nomenclature for meaning of unknowns.

$$T - T_o = 3 k T_o^2 w_2 M W_o / \sigma_o a_o M W_2 \quad (1)$$

$$d \ln A / d K = 1 / 6 K \quad (2)$$

The precursor (2) was shown to be true for all degrees of polymerization ($r=1$ to $r=\infty$) [see Refs. 2 and 3]. From Eq. (2), A and K are in Eq. (3), which is Eq. [15], one starting equation, in BLANDER/KATZ’S highly cited paper [see Ref. 4]. J is the nucleation rate, A the prefactor and K the exponent. Both A and K are complicated expressions.

$$J = A \exp(K) \quad (3)$$

There is another paper, which has three other starting equations [SIOW/PATTERSON, see Ref. 5: [1], [2] and [3] there]. Eq. (3) is for homogeneous nucleation of pure liquids from liquid--->gas. However, we envisage (3) in this paper as applying it in reverse by detailed balancing where the gaseous phase in the cloud is the mixture, species o, and the water is the solute, species 2.

II. RESULTS AND NOMENCLATURE

We have to assign meaning to the unknowns in (1) for the cloud to see if it can help in understanding the process as the cloud cools and rain drops form when the dew point is reached.

Nomenclature

a_o = surface area of water molecule at T_o (following Ref. 1)

k = Boltzmann constant

MW_o = average molecular weight of air (not including water)

MW₂ = molecular weight of water

MW_o and MW₂ drop out

n_0, n_2, n = moles of air (not water), water vapor moles, total gaseous moles

$P_{H_2O}, P^*_{H_2O}, P_{air}$ = vapor pressure of water, equilibrium water vapor pressure, air pressure at T

RH = relative humidity inside the cloud

T = temperature inside cloud (Kelvin)

T_0 = dew point temperature (Kelvin) – onset of rain

w_2 = weight fraction water vapor

σ_0 = surface tension of liquid water at T_0 (a water droplet is the new phase – following Ref. 1)

$$n_0 + n_2 = n \tag{4a}$$

$$w_2 = \frac{n_2 MW_2}{(n_0 MW_0 + n_2 MW_2)} \tag{4b}$$

We know $n_0 MW_0 \gg n_2 MW_2$ because the weight fraction of water, w_2 , in the atmosphere is about 2%. Then, we introduce n where $n \approx n_0$ and n = moles of atmospheric molecules in a given volume, and $MW_0 \approx MW_2$. Equation (1) then reduces to the following, which cannot be solved as a quadratic in T_0 , but actually has to be solved by a computer program because some variables depend on T and others on T_0 .

$$T - T_0 = (3 k T_0^2 / \sigma_0 a_0) (n_2/n) \tag{5}$$

In order to test (5), one has to have the relative humidity, RH, to get T_0 (dew point), and in order to get RH you need the value of (n_2/n) , which is the mole fraction of water vapor in the cloud. The author is offering (5) as a formula that might help understand rain clouds. The original data supporting Eq. (1) was from JENNINGS/MIDDLEMAN 1985 (See. Ref. 6).

The relative humidity, RH, is related to (n_2/n) in the following way.

$$RH \equiv \frac{\text{vapor pressure of water/equilibrium (saturated) vapor pressure of water}}{P_{H_2O}/P^*_{H_2O}} \tag{6}$$

Because the ideal gas law dictates that for a given volume and temperature P is proportional to n, viz. $PV = nRT$, we have the following for (n_2/n) .

$$n_2/n = \text{water vapor pressure/air pressure} = P_{H_2O}/P_{air} \tag{7}$$

Eliminating P_{H_2O} , we have the relation between RH and (n_2/n) .

$$RH = (n_2/n) / (P^*_{H_2O}/P_{air}) \tag{8}$$

Thus, knowing certain quantities gains you knowledge of RH and T_0 .

Combining (5) and (8) gives (9).

$$T - T_0 = (3 k T_0^2 / \sigma_0 a_0) (P^*_{H_2O}/P_{air}) RH \tag{9}$$

This is mathematically rigorous given that BLANDER/KATZ holds for gaseous solution to critical water droplets. In Pruppacher/Klett, pp 10-11 (See Ref. 7) the relative humidity inside clouds is near 100%, but that doesn't mean RH drops out but RH remains a measurable variable.

III. CONCLUSION

The dew point must be understood by homogeneous nucleation, as presented here. Indeed, Eq. (1) is a general one, so it can be applied to clouds as the water vapor approaches the supercooling limit. Maybe atmospheric chemists can determine whether Eq. (9) is useful in predicting rainfall.

IV. ACKNOWLEDGMENT

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