International Advance Journal of Engineering Research (IAJER)
Volume 4, Issue 03 (March- 2021), PP 16-33
ISSN: 2360-819X
www.iajer.com

Research Paper **Open Access**

A Theory of Composite Materials used forLiquefied Natural Gas (LNG) Tanks

Samuel W Chung*, University of Utah, USA
Hyun-ho Ju, Dong Guk University, South Korea

Abstract: Laminated composite materials can be a perfect set up when it is properly combined and formulated with proper theory of mechanics. That is in terms of the solid strength of materials and the thermal strength. Being LNG is of cryogenic temperature, thermal strength and insulation are vitally important compare to ordinary strength, such as tensile and shear stresses. By adopting proper combination of the materials of the tank, we can take the LNG tank wall pre-insulated and no additional insulation can be necessary. That is the vital advantage of the laminated composite materials. We have freedom of using as many layers as we need, as many different materials of the layer we want, also as thick of each layer and the total thickness as we want.

I. Introduction

A significant commercial application of cryogenics is the liquefaction, transport and storage of natural gas. Liquefied Natural Gas (LNG) is generally 95 percent methane with a few percent ethane and much lower concentrations of propane and butane. LNG liquefies at 111.6 K.

Figure 1 through 5 show schematic of an LNG fuel system for a maritime ferry "LNG systems for natural gas propelled ships,"

Unlike many applications of cryogenics, the motivation for using LNG is not the provision of lower temperatures but rather the very large volume reduction (greater than a factor of 600) between natural gas at atmospheric pressure and temperature and LNG.

Volume reduction allows for efficient transport of large amounts of natural gas. The natural gas industry typically uses LNG for sea transport, with regular shipments occurring between producing countries and consuming countries, for example between the Middle East and Japan or between North Siberiato Europe and Asia.

Once the LNG arrives at a consuming country it is typically converted to high pressure 300 K gas and distributed via pipeline. In some cases, further shipment within the consuming country is also carried out via LNG. Given the amounts of LNG shipped regularly, this is a major industry with a very large commercial value.

Due to its lower emissions upon combustion, both municipalities and corporate groups are increasingly using natural gas as a fuel for buses and other fleet vehicles. Generally, this is accomplished with room temperature compressed natural gas (CNG) but there are uses of LNG as a fuel in maritime ferries. Figure 1 shows an example of such a system.

LNG provides a rich field for cryogenic engineering. Given the amount of LNG transported, optimization of liquefaction plants is an ongoing effort, frequently using mixed gas refrigeration (Cold Facts Vol 32 No 1). Conversion of LNG back to 300 K gas is often carried out by heat exchange with ambient air or sea water, but work has been carried out to use the cold LNG to assist in air separation production of dry ice and freezing of food, thus improving the overall efficiency of the LNG industry.

Uniformly Valid Shell Theory of Hybrid Anisotropic Materials Applied to the LNG Tank

Here we adopted a cylindrical shell for the LNG storage.In the previous publications of the author listed in the References, we have formulated shell theories by using different combinations of length scales. Each theory implied unique physical characteristics which could be deduced from the order of magnitude of stress and displacement components, the stiffness matrix of the stress resultants and the governing equations.

We now wish to develop a uniformly valid shell theory which includes all the terms present in each of the previous theories. Since we are combining the theories together, we cannot nor need not use dimension-less coordinates any more, except for the radial coordinates. Each component of displacements, stresses and compliance matrix is also taken as the actual quantity.The elasticity equations written in terms of the dimensionless coordinate y are given by:

Stress-Displacement Relations

$$u_{r,y} = \lambda a[S_{11}\sigma_z + S_{12}\sigma_\theta + S_{13}\sigma_r + S_{14}\tau_{r\theta} + S_{15}\tau_{rz} + S_{16}\tau_{\theta z}] \quad (8.1)$$

$$u_{z,y} + \lambda a u_{r,z} = \lambda a\,[S_{51}\sigma_z + S_{52}\sigma_\theta + S_{53}\sigma_r + S_{54}\tau_{r\theta} + S_{55}\tau_{rz} + S_{56}\tau_{\theta z}]$$

$$\lambda u_{r,\theta} + (1+\lambda y)u_{\theta,y} - \lambda u_\theta = \lambda a(1+\lambda y)[S_{41}\sigma_z + S_{42}\sigma_\theta + S_{43}\sigma_r + S_{44}\tau_{r\theta} + S_{45}\tau_{rz} + S_{46}\tau_{\theta z}]$$

Equilibrium Equations

$$[(1+\lambda y)\tau_{r\theta}]_{,y} + \lambda\sigma_{\theta,\theta} + \lambda a\tau_{\theta z,z} + \lambda\tau_{r\theta} = 0$$

$$[(1+\lambda y)\tau_{rz}]_{,y} + \lambda\tau_{\theta z,\theta} + \lambda a[(1+\lambda y)\sigma_z]_{,z} = 0 \quad (8.2)$$

$$[(1+\lambda y)\sigma_r]_{,y} + \lambda\tau_{r\theta,\theta} + \lambda a\tau_{rz,z} - \lambda\sigma_\theta = 0$$

The equations of the uniformly valid first approximation are determined from equations (8.1) and (8.2) by keeping only those terms found necessary in the various previous first approximation theories. As certain second approximation terms were necessary to obtain the first approximation, these must also be kept. The resulting equations for the displacements and in-plane stresses become,

$$u_{r,y} = 0$$

$$u_{z,y} + \lambda a u_{r,z} = 0 \quad (8.3)$$

$$\lambda u_{r,\theta} + (1+\lambda y)u_{\theta,y} - \lambda u_\theta = 0$$

$$u_{z,z} = S_{11}\sigma_z + S_{12}\sigma_\theta + S_{16}\tau_{\theta z}$$

$$(1/a)(u_{\theta,\theta} + u_r) = S_{21}\sigma_z + S_{22}\sigma_\theta + S_{26}\tau_{\theta z}$$

$$(1+\lambda y)u_{\theta,z} + (1/a)u_{z,\theta} = S_{61}\sigma_z + S_{62}\sigma_\theta + S_{66}\tau_{\theta z}$$

The integration with respect to y can now be carried out in the same fashion as was done in the previous chapters. Integration of the first three equations yields the displacements. They are:

$$u_r = U_r(z,\theta)$$

$$u_\theta = (1+\lambda y)U_\theta(z,\theta) - \lambda U_{r,\theta}\, y \qquad (8.4)$$

$$u_z = U_z(z,\theta) - a\lambda U_{r,z}\, y$$

where **Ur , U$_\theta$, Uz** are the **y=0** surface displacement components. Note here that the radial displacement is independent of the thickness coordinate while the circumferential and longitudinal displacements are of linear dependence. The theory thus incorporates the hypothesis of the preservation of the normal. Substitution of results (8.4) into the next three equations yields the in-plane stress-strain relations:

$$\begin{Bmatrix} \sigma_z \\ \sigma_\theta \\ \tau_{\theta z} \end{Bmatrix} = [\,C\,] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} + [C] \begin{Bmatrix} K_1 \\ K_2 \\ K_{12} \end{Bmatrix} \lambda a y \qquad (8.5)$$

where [C] is the symmetric matrix given by

$$[\,C\,] = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix}^{-1} \qquad (8.6)$$

In the above the **ε'$_s$**are defined by

$$\varepsilon_1 = U_{z,z}$$

$$\varepsilon_2 = (1/a)(U_r + U_{\theta,\theta}) \tag{8.7}$$

$$\varepsilon_{12} = U_{\theta,z} + (1/a)U_{z,\theta}$$

and the K's.the changes of curvature by

$$K_1 = - U_{r,zz}$$

$$K_2 = -(1/a^2)(U_{r,\theta\theta} - U_{\theta,\theta}) \tag{8.8}$$

$$K_{12} = -(2/a)(U_{r,\theta z} - U_{\theta,z})$$

We note here that in equations (8.5) we have kept only terms linear in y as it was indicated we should do by the results of the preceding theories. As we mentioned earlier, the curvature terms (8.8) play the role of correction to the y=0 strains for points away from the inner surface of the shell. The strains of the inner surface are given by (8.7).

The equilibrium equations of this theory which contains all the terms existing in the previous theories are:

$$[(1+\lambda y)\tau_{r\theta}]_{,y} + \lambda\sigma_{\theta,\theta} + a\lambda\tau_{\theta z,z} + \lambda\tau_{r\theta} = 0 \qquad (8.9)$$

$$\tau_{rz,y} + \lambda\tau_{\theta z,\theta} + a\lambda\sigma_{z,z} = 0$$

$$[(1+\lambda y)\sigma_r]_{,y} + \lambda\tau_{r\theta,\theta} + a\lambda\tau_{rz,z} - \lambda\sigma_\theta = 0$$

Substitution of relations (8.5) into the equilibrium equations and carrying out the integration with respect to y yields:

$$\begin{aligned}\tau_{rz} = T_{rz} &- \lambda[(A_{13}\varepsilon_{1,\theta} + A_{23}\varepsilon_{2,\theta} + A_{33}\varepsilon_{12,\theta}) \\ &+ a\lambda(B_{13}K_{1,\theta} + B_{23}K_{2,\theta} + B_{33}K_{12,\theta})] \\ &- a\lambda[(A_{11}\varepsilon_{1,z} + A_{12}\varepsilon_{2,z} + A_{13}\varepsilon_{12,z}) \\ &+ a\lambda(B_{11}K_{1,z} + B_{12}K_{2,z} + B_{13}K_{12,z})]\end{aligned} \qquad (8.10)$$

$$\begin{aligned}\tau_{r\theta} = T_{r\theta} &- \lambda[(A_{12}\varepsilon_{1,\theta} + A_{22}\varepsilon_{2,\theta} + A_{23}\varepsilon_{12,\theta}) \\ &+ a\lambda(B_{12}K_{1,\theta} + B_{22}K_{2,\theta} + B_{23}K_{12,\theta}) \\ &+ a\lambda^2(F_{22}K_{1,\theta} + F_{23}K_{12,\theta})] \\ &- a\lambda[(A_{13}\varepsilon_{1,z} + A_{23}\varepsilon_{2,z} + A_{33}\varepsilon_{12,z}) \\ &+ a\lambda[(B_{13}K_{1,z} + B_{23}K_{2,z} + B_{33}K_{12,z})]\end{aligned} \qquad (8.11)$$

$$\sigma_r = T_r + [(A_{12}\varepsilon_1 + A_{22}\varepsilon_2 + A_{23}\varepsilon_{12}) + a\lambda(B_{12}K_1 + B_{22}K_2 + B_{13}K_3)] \tag{8.12}$$

$$- a\lambda[T_{rz,z}y - a\lambda^2(E_{13}K_{1,\theta z} + E_{23}K_{2,\theta z} + E_{33}K_{12,\theta z})$$

$$-a^2\lambda^2(E_{11}K_{1,zz} + E_{12}K_{2,zz} + E_{12,zz})]$$

$$-\lambda[T_{r\theta,\theta}y - a\lambda^2(E_{12}K_{1,\theta\theta} + E_{22}K_{2,\theta\theta} + E_{23}K_{12,\theta\theta})$$

$$-a^2\lambda^2(E_{13}K_{1,\theta z} + E_{23}K_{2,\theta z} + E_{33}K_{12,\theta z})]$$

Satisfaction of the conditions at the inner and outer surface of the shell (2.7) yields

$$T_{rz} = T_{r\theta} = T_r = 0 \tag{8.13}$$

and

In the derivation of (8.14-8.16) we have omitted all terms which the preceding analysis showed to be of higher order.

On substituting the in-plane stresses given by (8.5) into the relations (2.8) we obtain the following expressions for the stress resultants:

$$\underline{A}_{11}aU_{z,zz} + 2\underline{A}_{13}U_{z,\theta z} + \underline{A}_{33}(1/a)U_{z,\theta\theta} + \underline{A}_{13}aU_{\theta,zz} + (\underline{A}_{12} + \underline{A}_{33})U_{\theta,\theta z} + \underline{A}_{23}(1/a)U_{\theta,\theta\theta}$$

$$+\underline{A}_{12}U_{r,z} + \underline{A}_{23}(1/a)U_{r,\theta} - a\lambda[\underline{B}_{11}aU_{r,zzz} + 3\underline{B}_{13}U_{r,zz\theta} + (1/a)(\underline{B}_{12} + 2\underline{B}_{33})U_{r,z\theta\theta}$$

$$+\underline{B}_{23}(1/a^2)U_{r,\theta\theta\theta} + \underline{B}_{23}(1/a^2)U_{r,\theta} + (1/a)(\underline{B}_{12} + 2\underline{B}_{33})U_{r,z} \tag{8.14}$$

$$- 2\underline{B}_{13}U_{\theta,zz}] = 0$$

$$\underline{A}_{13} a U_{z,zz} + (\underline{A}_{12}+\underline{A}_{33}) U_{z,\theta z} + \underline{A}_{23}(1/a)U_{z,\theta\theta} + \underline{A}_{22}(1/a)U_{\theta,\theta\theta} + \underline{A}_{33} a U_{\theta,zz} +$$

$$2\underline{A}_{23}U_{\theta,\theta z} + \underline{A}_{23}U_{r,z} + \underline{A}_{22}(1/a)U_{r,\theta} - \qquad (8.15)$$

$$a\lambda[\underline{B}_{13} a U_{r,zzz} + (\underline{B}_{12}+2\underline{B}_{33})U_{r,\theta zz} + 3\underline{B}_{23}(1/a)U_{r,\theta\theta z} + \underline{B}_{22}(1/a^2)U_{r,\theta\theta\theta} +$$

$$2\underline{B}_{23}(1/a)U_{r,z} + \underline{B}_{22}(1/a^2)U_{r,\theta}] - a\lambda^2\Big[(2\underline{F}_{23}(1/a)U_{r,\theta\theta z} + \underline{F}_{22}(1/a^2)U_{r,\theta\theta\theta} -$$

$$- 2\underline{F}_{23}(1/a)U_{\theta,\theta z} - \underline{F}_{22}(1/a^2)U_{\theta,\theta\theta})\Big] = 0$$

a : Inside Radius of Cylindrical Shell
h : Total Thickness of the Shell Wall
d; Distance (thickness) from Inside Radiusand to mechanical neutral surface
Si : Radius of Each Layer of Wall (I = 1, 2, 3 -- - to the number of layer)
L : Longitudinal Length Scale to be defined, Also Actual Length of the Cylindrical Shell
Π : Circumferential Length Scale of cylindrical shell to be defined
Ei : Young's Moduli in I Direction
Gij : Shear Moduli in i-j Face
Sij : Compliance Matrix of Materials of Each Layer
r : Radial Coordinate
Π : Circumferential Length Scale to be defined
Ɣ : Angle of Fiber Orientation
σ : Normal Stresses
ε : Normal Strains
z, θ, r : Generalized Coordinates in Longitudinal, Circumferential and Radial Directions Respectively
τ : Shear Stresses
εij : Shear Strains in i-j Face
λ ; Shell Thickness / Inside Radius (h/a)
Cij : Elastic Moduli in General
X, φ, Y : Non Dimensional Coordinate System in Longitudinal, Circumferential and Radial Directions Respectively

Table 1 List of Symbols

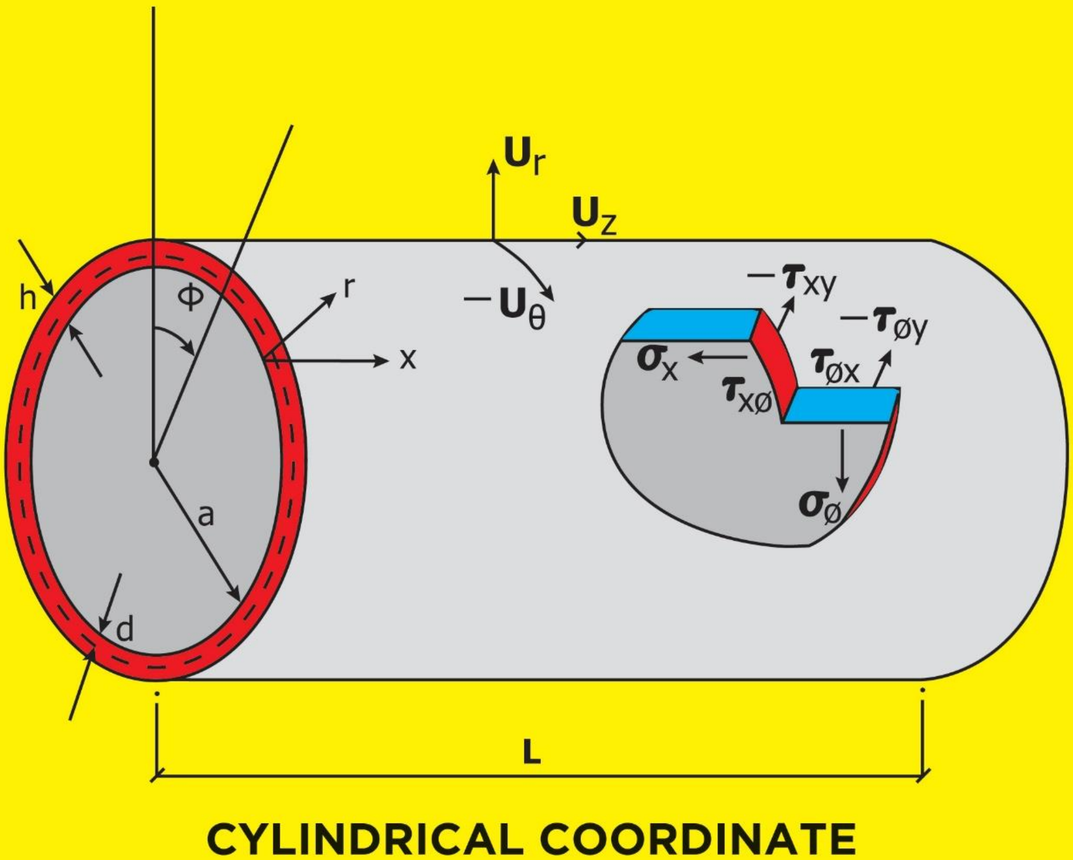

FIGURE 1 A CYLINDRICAL SHELL SHOWING DIMENSIONS, DEFORMATIONS AND STRESSES

$$\underline{A}_{12}U_{z,z}+\underline{A}_{22}(U_{r}+U_{\theta,\theta})/a+\underline{A}_{23}(U_{\theta,z}+U_{z,\theta}/a)$$

$$-a\lambda[\underline{B}_{12}U_{r,zz}+\underline{B}_{22}(U_{r,\theta\theta}+U_{r})/(a^2)+2\underline{B}_{13}(U_{r,\theta z}-U_{\theta,z})/a] \qquad (8.16)$$

$$+a\lambda[a\underline{D}_{11}U_{z,zzz}+3\underline{D}_{13}U_{z,zz\theta}+(1/a)(\underline{D}_{12}+2\underline{D}_{33})U_{z,\theta\theta z}+(1/a)\underline{D}_{23}U_{z,\theta\theta\theta}+$$

$$+a\underline{D}_{13}U_{\theta,zzz}+(\underline{D}_{12}+2\underline{D}_{33})U_{\theta,zz\theta}+(3/a)\underline{D}_{23}U_{\theta,\theta\theta z}+(1/a^2)\underline{D}_{22}U_{\theta,\theta\theta\theta}+$$

$$+\underline{D}_{12}U_{r,zz}+2\underline{D}_{23}(1/a)U_{r,\theta z}+(\underline{D}_{22}/a^2)U_{r,\theta\theta}]$$

$$-a^2\lambda^2[\underline{E}_{13}U_{r,\theta zzz}+(\underline{E}_{23}/a^2)U_{r,\theta\theta\theta z}+2(\underline{E}_{33}/a)U_{r,\theta\theta zz}]$$

$$-a^3\lambda^2[\underline{E}_{11}U_{r,zzzz}+(\underline{E}_{12}/a^2)U_{r,\theta\theta zz}+\underline{E}_{13}(2/a)U_{r,\theta zzz}]$$

$$-a\lambda^2[\underline{E}_{12}U_{r,\theta\theta zz}+(\underline{E}_{22}/a^2)(U_{r,\theta\theta\theta\theta}+U_{r,\theta\theta})+2(\underline{E}_{23}/a)U_{r,\theta\theta\theta z}]$$

$$-a^2\lambda^2[\underline{E}_{13}U_{r,zzz\theta}+(\underline{E}_{23}/a^2)U_{r,\theta\theta\theta z}+2(\underline{E}_{33}/a)U_{r,\theta\theta zz}] = p$$

$$\begin{Bmatrix} N_z \\ N_\theta \\ N_{z\theta} \\ N_{\theta z} \\ M_z \\ M_\theta \\ M_{z\theta} \\ M_{\theta z} \end{Bmatrix} = \left[\begin{array}{c|c} \overline{A} & \overline{B} \\ \hline \overline{B} & \overline{D} \end{array}\right] \begin{Bmatrix} \epsilon_{1d} \\ \epsilon_{2d} \\ \epsilon_{12d} \\ K_1 \\ K_2 \\ K_{12} \end{Bmatrix} \qquad (8.17)$$

where

$$\varepsilon_{1d} = \varepsilon_1 + dK_1$$

$$\varepsilon_{2d} = \varepsilon_2 + dK_2 \qquad (8.18)$$

$$\varepsilon_{12d} = \varepsilon_{12} + dK_{12}$$

and

$$[\,\overline{A}\,] = \begin{bmatrix} [\lambda a/(1+d/a)][\underline{A}_{11},\ \underline{A}_{12},\ \underline{A}_{13}] \\ (\lambda a)\ [\underline{A}_{21},\ \underline{A}_{22},\ \underline{A}_{23}] \\ [\lambda a/(1+d/a)][\underline{A}_{31},\ \underline{A}_{32},\ \underline{A}_{33}] \\ (\lambda a)\ [\underline{A}_{31},\ \underline{A}_{32},\ \underline{A}_{33}] \end{bmatrix} \qquad (8.19)$$

$$[\,\overline{B}\,] = \begin{bmatrix} [\lambda a/(1+d/a)][(\lambda a\underline{B}_{11}-d\underline{A}_{11}),(\lambda a\underline{B}_{12}-d\underline{A}_{12}),(\lambda a\underline{B}_{13}-d\underline{A}_{13})] \\ (\lambda a)\ [\ (\lambda a\underline{B}_{21}-d\underline{A}_{21}),(\lambda a\underline{B}_{22}-d\underline{A}_{22}),(\lambda a\underline{B}_{23}-d\underline{A}_{23})] \\ [\lambda a/(1+d/a)][(\lambda a\underline{B}_{31}-d\underline{A}_{31}),(\lambda a\underline{B}_{32}-d\underline{A}_{32}),(\lambda a\underline{B}_{33}-d\underline{A}_{33})] \\ (\lambda a)\ [\ (\lambda a\underline{B}_{31}-d\underline{A}_{31}),(\lambda a\underline{B}_{32}-d\underline{A}_{32}),(\lambda a\underline{B}_{33}-d\underline{A}_{33})] \end{bmatrix}$$

$$[D] = \begin{bmatrix} [\lambda^2a^2/(1+d/a)][(\lambda a\underline{F}_{11}-2d\underline{B}_{11}+\frac{d^2}{\lambda a}\underline{A}_{11}),\ (\lambda a\underline{F}_{12}-2d\underline{B}_{12}+\frac{d^2}{\lambda a}\underline{A}_{12}), \\ \qquad (\lambda a\underline{F}_{13}-2d\underline{B}_{13}+\frac{d^2}{\lambda a}\ \underline{A}_{13}) \\ (\lambda^2a^2)\ \ [(\lambda a\underline{F}_{21}-2d\underline{B}_{21}+\frac{d^2}{\lambda a}\ \underline{A}_{21}),\ \ (\ldots),\ \ (\ldots) \\ [\lambda^2a^2/(1+d/a)][(\lambda a\underline{F}_{31}-2d\underline{B}_{31}+\frac{d^2}{\lambda a}\underline{A}_{31}),\ (\ldots),\ (\ldots) \\ (\lambda^2a^2)[\ \ (\lambda a\underline{F}_{31}-2d\underline{B}_{31}+\frac{d^2}{\lambda a}\ \underline{A}_{31}),\ (\ldots),\ (\ldots) \end{bmatrix}$$

Equations (8.4), (8.5) and (8.10 – 8.16) represent the equations of the uniformly valid first approximation theory. Since all of the effects found necessary in each of the previous theories are present in this theory, it should be sufficient to treat all shell boundary value problems as a first approximation.

Application

Strain-Displacement Relations

$$\varepsilon_1 = U_{z,z} \tag{9.35}$$

$$\varepsilon_2 = U_r/a$$

$$\varepsilon_{12}=U_{\theta,z}$$

$$K_1 = -U_{r,zz}$$

$$K_2 = 0$$

$$K_{12}= 2U_{\theta,z}/a$$

Governing Equations

$$\underline{A}_{11}aU_{z,zz}+\underline{A}_{13}aU_{\theta,zz}+\underline{A}_{12}U_{r,z}-a\lambda[\underline{B}_{11}aU_{r,zzz}+(\underline{B}_{12}+2\underline{B}_{33})(1/a)U_{r,z} \tag{9.36}$$

$$-2\underline{B}_{13}U_{\theta,zz}] = 0$$

$$\underline{A}_{13}aU_{z,zz}+\underline{A}_{33}aU_{\theta,zz}+\underline{A}_{23}U_{r,z}-a\lambda[\underline{B}_{13}aU_{r,zzz}+2\underline{B}_{23}(1/a)U_{r,z}] = 0$$

$$\underline{A}_{12}U_{z,z}+\underline{A}_{22}(1/a)U_r+\underline{A}_{23}U_{\theta,z}- a\lambda[\underline{B}_{12}U_{r,zz}+\underline{B}_{22}(1/a^2)U_r-2\underline{B}_{13}(1/a)U_{\theta,z}]$$

$$-a^3\lambda^2\underline{E}_{11}U_{r,zzzz}+a\lambda(a\underline{D}_{11}U_{z,zzz}+a\underline{D}_{13}U_{\theta,zzz}+\underline{D}_{12}U_{r,zz}) = p$$

The first two equations of (9.36) give us the following relations for U_z and U_θ
:

$$U_{\theta,z} = \mathbf{R}a\lambda U_{r,zz} - \mathbf{S}\frac{1}{a}U_r + C_1^* \tag{9.37}$$

$$U_{z,z} = \mathbf{T}a\lambda U_{r,zz} - \mathbf{W}\frac{1}{a}U_r + C_2^*$$

where C_1* and C_2* are the constants of integrations and the coefficients R, S, T, W aredefined as follows:

$$\mathbf{R} = (\underline{B}_{13}\underline{A}_{11} - \underline{B}_{11}\underline{A}_{13})/\psi \qquad (9.38)$$

$$\mathbf{S} = [\lambda\underline{A}_{13}(\underline{B}_{12}+2\underline{B}_{33}) + \underline{A}_{11}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{12} - 2\lambda\underline{B}_{23}\underline{A}_{11}]/\psi$$

$$\mathbf{T} = (\underline{B}_{11}\underline{A}_{33} - \underline{A}_{13}\underline{B}_{13} - 2\lambda\underline{B}^2_{13})/\psi$$

$$\mathbf{W} = [\underline{A}_{33}\underline{A}_{12} - \lambda\underline{A}_{33}(\underline{B}_{12}+2\underline{B}_{33}) - \underline{A}_{13}\underline{A}_{23} - 2\lambda\underline{B}_{13}\underline{A}_{23} + 2\lambda\underline{B}_{23}\underline{A}_{13} + 4\lambda^2\underline{B}_{13}\underline{B}_{23}]/\psi$$

and

$$\psi = \underline{A}_{11}\underline{A}_{33} - \underline{A}^2_{13} - 2\lambda\underline{B}_{13}\underline{A}_{13} \qquad (9.39)$$

On substituting (9.38) into the third equation of (9.36) we obtain a differential equation for U only,

$$N_1 U_{r,zzzz} - 2N_2 U_{r,zz} + N_3 U_r = p - N_4 \qquad (9.40)$$

where

$$N_1 = a^3\lambda^2(\underline{D}_{13}\mathbf{R} + \underline{D}_{11}\mathbf{T} - \underline{E}_{11}) \qquad (9.41)$$

$$N_2 = a\lambda[\underline{B}_{12} - \underline{D}_{12} - \mathbf{T}\underline{A}_{12} - \mathbf{R}(\underline{A}_{23}+2\lambda\underline{B}_{13}) + \underline{D}_{13}\mathbf{S} + a^2\lambda\underline{D}_{11}\mathbf{W}]/2$$

$$N_3 = [-\mathbf{W}\underline{A}_{12} - \mathbf{S}(\underline{A}_{23}+2\lambda\underline{B}_{13}) + \underline{A}_{22} - \lambda\underline{B}_{22}]/a$$

$$N_4 = \underline{A}_{12}C^*_2 + (\underline{A}_{23}+2\lambda\underline{B}_{13})C^*_1$$

The equation (9.34) is similar in appearance to the governing equations for other theories given by equations (9.19) and (9.24) and so is the homogeneous solution. The homogeneous solution can be written as

$$U^H_r = \exp(-N_5 z)(A_1\cos N_6 z + A_2\sin N_6 z) + \exp(-N_5\eta)(A_3\cos N_6\eta + A_4\sin N_6\eta) \qquad (9.42)$$

where A_i (i = l, 2, 3, 4) are the constants of integration to be determined andN_5, N_6 are as defined previously. As we are now dealing with actual coordinates (except for the thickness coordinate), we define n as follows:

$$\eta = L - z \qquad (9.43)$$

where L is the actual length of the cylinder. The constants of integration are to be determined from the following edge conditions:

$$U_r = U_{r,z} = U_z = U_\theta = 0 \qquad (z = 0,\ y = d/h) \qquad (9.44)$$

$$U_r=0,\ N_z=N,\ M_z=N(H-d),\ T=2\pi a(1+d/a)[M_{z\theta}+a(1+d/a)N_{z\theta}]$$

$$(z = L,\ y = d/h)$$

The particular solution of (9.34) is

$$U_r^p = (p - N_4)/N_3 \qquad (9.45)$$

and the complete solution of equation (9.34) is then given by

$$U_r = U_r^p + U_r^H \qquad (9.46)$$

Having obtained the above solutions for each of the theories, numerical calculations are now carried out for a shell of the following dimensions:

We thus have a thickness to radius ratio of

$$\lambda = 0.025 \qquad (9.48)$$

Each of the layers is taken to be .025 in. thick and thus the dimensionless distances from the bottom of the first layer are given by

$$s_1=0.,\ s_2=0.25,\ s_3=0.5,\ s_4=0.75,\ s_5=1.0 \qquad (9.49)$$

As mentioned previously, each layer of the symmetric angle ply configuration (elastic symmetry axes y are oriented at (+γ , -γ, - γ, + γ) is taken to be orthotropic with engineering elastic coefficients representing those for a boron/epoxy material system,

$$E_1 = 35\text{x}10^6 \text{ psi}, \qquad E_2 = 2.75\text{x}10^6 \text{ psi} \qquad (9.50)$$

$$G_{12} = 0.75\text{x}10^6 \text{ psi}, \qquad \nu = 0.25$$

Herethedirection γsignifies the direction parallel to the fibers while 2 is the transverse direction. Angles chosen were = 0 , 15, 30, 45 and 60. Use ofγ the results yields the mechanical properties for the different symmetric angle ply configurations.
We next apply the following edge loads:

$$N = p \qquad (9.51)$$

and take

$$\sigma = p/\lambda \tag{9.52}$$

$$H = (3/4)h$$

Shown in Figures are the variation of the dimensionless radial displacement with the actual distance along the axis for the different theories. The reference surface for the chosen configuration is given by

$$d/h = 1/2 \tag{9.53}$$

As mentioned above, the theory associated with length scales a is a membrane type theory and it's radial displacement is a function of the dimension less pressure and the two integration constants determined from the edge conditions (9.12). As the Figures demonstrate the radial displacement of this theory is constant over the entire length of the shell. The variation of the magnitude of radial displacement due to the change of cross-ply angle is almost identical compared to the other theories except for the fact that the theory can not describe the deformation pattern due to the boundary conditions while the other theories showing the radial deformations of the so-called edge effect zone.The theory associated with longitudinal length scale $(ah)^{1/2}$ and circumferential length scale a is similar to the axi-symmetric version of the theoryof length scales $(ah)^{1/2}$ in the following aspects:

a) Expressions for strains and curvatures are identical as they wereshown in (9.14) and (9.26). This is due to the fact that the theoryof length scales $(ah)^{1/2}$ are much simplified by the axi-symmetricproperty while the other theory is closer in fashion to the axi-symmetric behavior by it's nature because of the larger circumferential length scale we used for the theory, i.e. a.

b) Although the expressions for the particular solutions are different as indicated in (9.24) and (9.33), the combined form of governing equations and the homogeneous solutions, as shown in (9.20) and (9.32),are identical in form. This is due to the same length scales beingused in longitudinal direction, $(ah)^{1/2}$, for both theories and again,the axial symmetry.In obtaining the homogeneous solutions for both theories, it was assumedthat the cylinder has such material properties and geometric dimensions so asto justify the decay type solutions (9.20) and (9.32). In order to havethese decay type solutions we first must have that the value of term as shown in (9.22) must be real.

$$N_1 N_3 - N_2^2 > 0 \tag{9.54}$$

Secondly, the dimensionless shell length (must be sufficiently largercompared to the axial length scale used in the basic formulation of the theories, $(ah)^{1/2}$, so that interaction effects from the opposite edges may beneglected. The condition for satisfying this can be obtained by comparingthe two decay terms in equation (9.20), exp ($-N_5$ x) and exp ($-N_5 5\,\zeta$).

This leads to

$$l > \Gamma \tag{9.55}$$

where

$$\Gamma = \pi (ah)^{1/2} (N_1/N_3)^{1/4} \tag{9.56}$$

The restriction of the cylinder length **l** to be larger than **r** is important in the analysis of cylindrical shells due to the difference, of nature of the solution. For the cylinder shorter than r, the edge conditions havean effect on each other and the solution is no longer of the decay type. Edge conditions in this case govern the deformation pattern as well as the magnitude. A short cylinder under external pressure and closed at both ends deform axi-symmetrically and can be considered a typical example of a problem where the solution has a decay length shorter than r. It must be noted here that unlike for isotropic homogeneous shells, the decay length **r** depends not only the shell geometry, h and a, but also on the material properties of each laminate.

As stated previously, contents and references show that the radial displacement of the shell at distances from the edge greater than r, from now on called the edge effect boundary layer, is nearly identical for both theories and closein magnitude to that of the solution which is obtained for length scales a. This is because, in the regions away from, the particular part of the solutions of governing equations dominate while the homogeneous solutions are more important within the boundary layer regions.

Because the results shown in the figures are nearly identical for the problem considered, no numerical calculations of the uniformly valid solution given by (9.46) is carried out.
It is also seen that wide variations in the magnitude of radial displacement take place with change in the cross-ply angle. The maximum displacement occurs at γ=30 degree while the minimum displacement is atγ = 60 degree. In each case, the displacements increase with increase in uγ up toγ = 30 degree and thereafter decrease. Figs. 4 and 5 show that the edge effect is sharper for small angle Gthan for larger ones. Similarly, deeper penetration of the edge effect is shown for small angles γ while weak and smooth edge effects are the case for large cross-ply angles .Also shown are the dimensionless displacement of an isotropic material of elastic coefficient $30x10^6$ psi and in Figs 8 - 1 0 are of single layer boron/epoxy composite we used for four layers case.

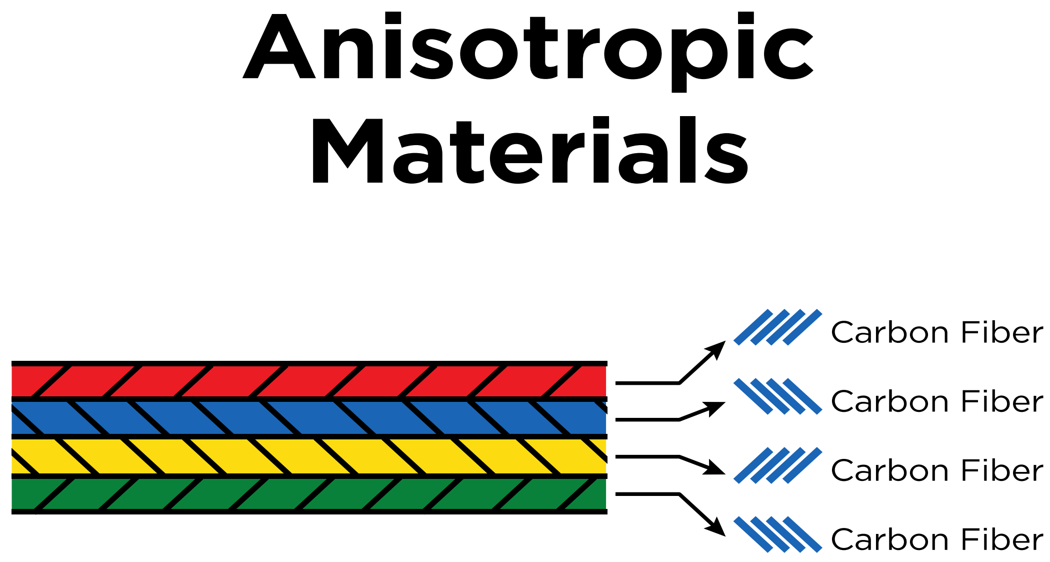

Figure 2 Combination of Materials

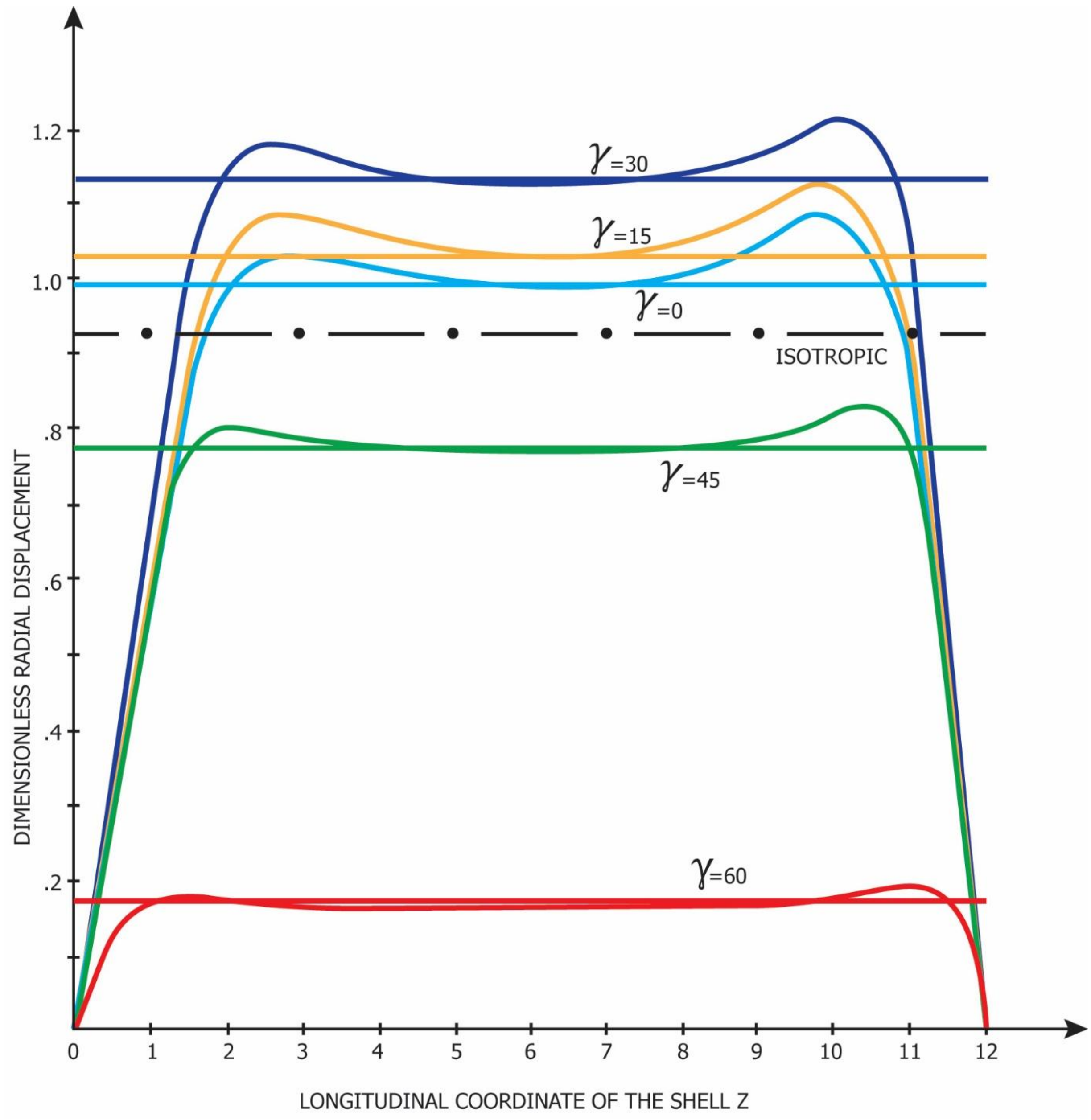

Figure 3 Radial Deformation

Conclusion

The freedom of challenge and the freedom of material choice were mathematically presented. It is great that we had invented composite materials being used for cryogenic tanks such as LNG, LOX and LHG tanks. Also the use can be extended to the space vehicles and structures for skyscrapers.However, care must be taken that all those material advantage can only be achieved by understanding and formulating the theories of physics and mechanics. LNG as well as nuclear reactors are super energy sources, we must use safely and composite materials can provide the answer as long as we continue our research for the materials. By utilizing the composite materials,

Double wall tanks for providing insulation in between the inner and outer wall, such as perlite, are no longer necessary.

Also the shell wall by itself can be super insulation as well as serving as good storage tank

Acknowledgement

The research was sponsored by Summit Partners in Menlo Park, California, USA, and is graciously acknowledged.

Reference

[1] DYM, C. L. and HOFF, N. J., "Pertubation-solutions for the bucklingproblem of axially compressed thin cylindrical shells", AFO SR-66--1755, SUDAAR No.282, Stanford University.

[2] BERT, C. W., "Structural theory for laminated anisotropic elasticshells", J. COMPOSITE MATERIALS 1 , 414 (1967).

[3] VICARIO, A. A. and RIZZO, R. R., "Effect of length on laminatedthin tube under combined loadings", J. COMPOSITE MATERIALS 4, 273 (1970).

[4] Chung, S.W., Hong, S.G. and Ju, G.S., "Details of Semi-Membrane Shell Theory of Hybrid Anisotropic Materials" International Journal of Composite Materials 2018.8(3):47-56,

[5] Chung, S.W., Hong, S.G., Ju, G.S. "Pure Membrane, Pseudo Membrane, and Semi Membrane Shell Theories of Hybrid Anisotropic Materials", Journal of Material Science and Engineering A 8 (5-6) (2018) 121-135,

[6] Chung, S. W.[1], Hong S. G.[2], Ju G. S.[3,] Semi-Membrane and Effective Length Theory of hybrid Anisotropic Materials, International Journal of Composite Materials. volume 7 (3), 2017: Contact Us: editor@sapub.org , ID:110900221,

[7] Chung, S. W.[1], Hong S. G.[2], Ju G. S.[3,] "Applications of Pure Membrane, Pseudo Membrane, and Semi Membrane Shell Theories of Hybrid Anisotropic Materials", International Journal of Composite Materialsp-ISSN: 2166-479X e-ISSN: 2166-4919,2018; 8(4): 73-90doi:10.5923/j.cmaterials.20180804.01

[8] Chung S. W.[1] , Hong, S. G.[2], Ju G. S.[3,,] "A Spherical Shell Theory of Hybrid Anisotropic Materials". International Journal of Composite Materialsp-ISSN: 2166-479X e-ISSN: 2166-49192018; 8(4): 97-104, ID:110900262, doi:10.5923/j.cmaterials.20180804

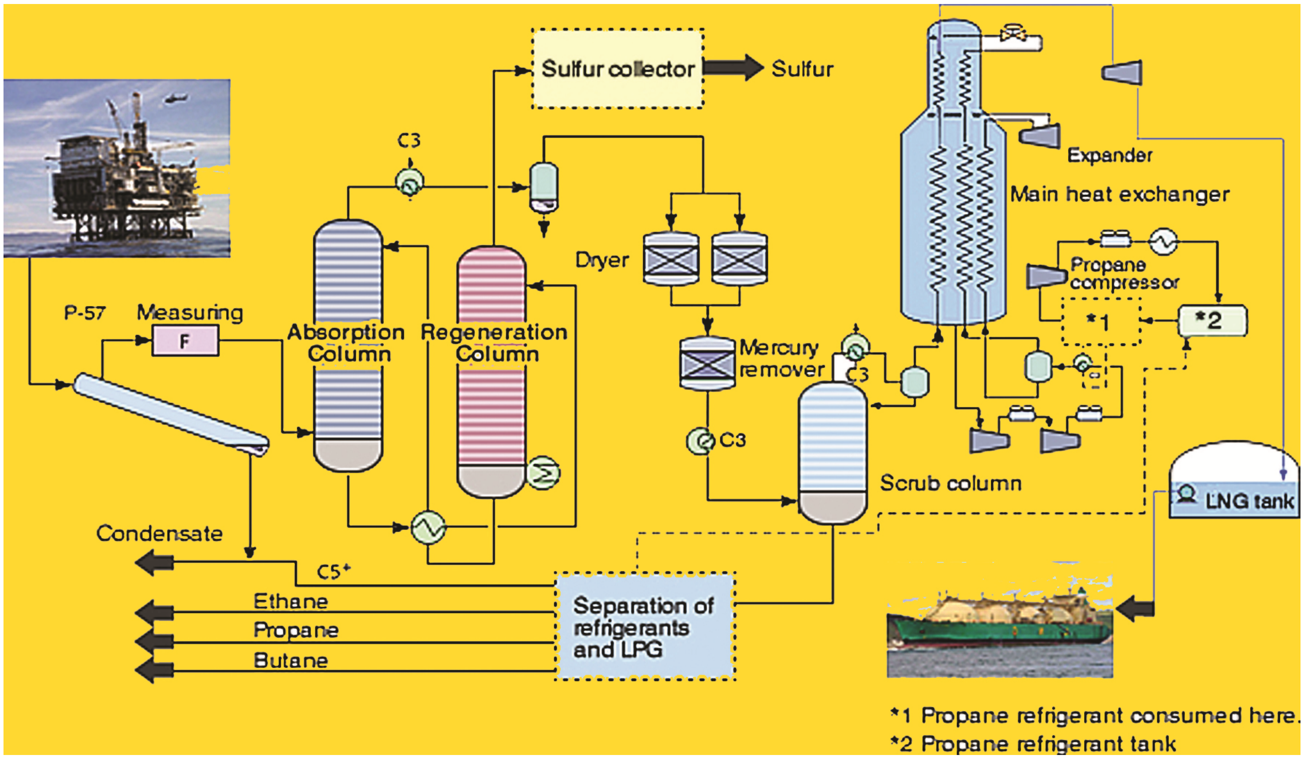

Figure 4 Example of a System

Figure 5