International Advance Journal of Engineering Research (IAJER)
Volume 3, Issue 6 (June- 2020), PP 09-24
ISSN: 2360-819X
www.iajer.com

Research Paper **Open Access**

Operation and Maintenance of HVAC System for Corona Virus

[1]Samuel W Chung, [2]Jeong Je Jo
[1]University of Utah Salt Lake City, Utah U S A
[2]E & S Solution Tae Gu City South Korea
[*]Corresponding Author: [1]Samuel W Chung

ABSTRACT:- Corona Virus is deadly spread out to thousands of healthy people and so strong as to infect to the surrounding people after people. We must keep the deadly virus indoor as much as we can. As have mentioned above, the hospitals are nothing but a pressure vessel with negative pressure compare to the atmospheric pressure
How? We must not allow the indoor air already contaminated to leak out to the atmosphere, which will transmit to other people. The new virus is so strong and so fast to transmit, it will spread within a few seconds to thousands of people and became patients immediately. To solve the problem, we must build a pressure vessel to keep the virus inside the vessel and not to leak outside. It should be a negative pressure compare to the atmospheric pressure. Another words, is to keep the indoor pressure lower than atmospheric pressure, so the contaminated indoor pressure is lower than outdoor pressure. It is a part of a traditional HVAC operation. This article shows step by step procedures of HVAC system design procedures, with emphasis of the air duct system design. Care must be taken to keep close attention to the maintenance of the system.

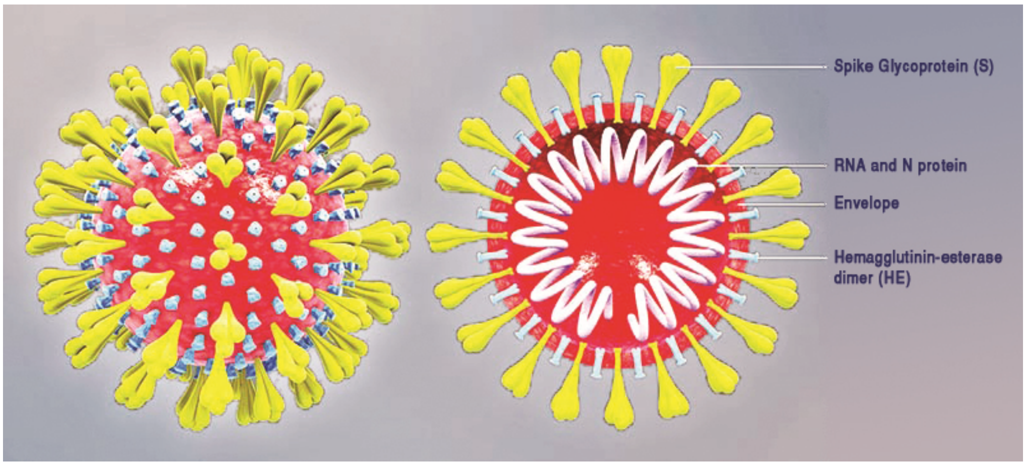
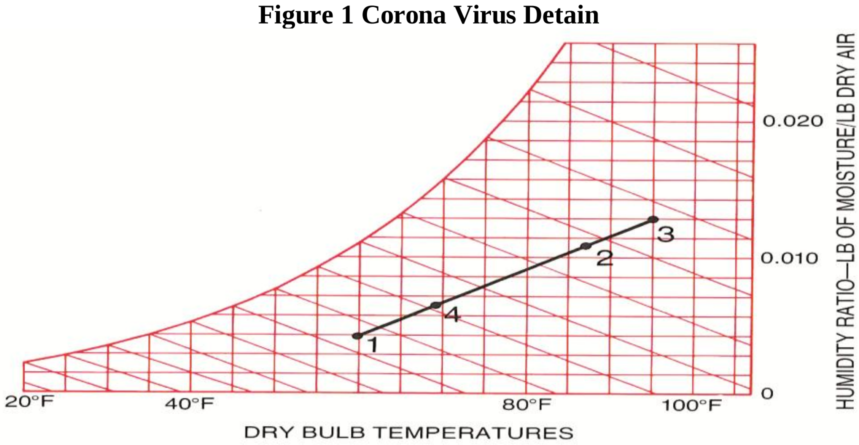


Figure 1 Corona Virus Detain



TOTAL HEAT TRANSFER PROCESS

Figure 2 Psychrometric Chart

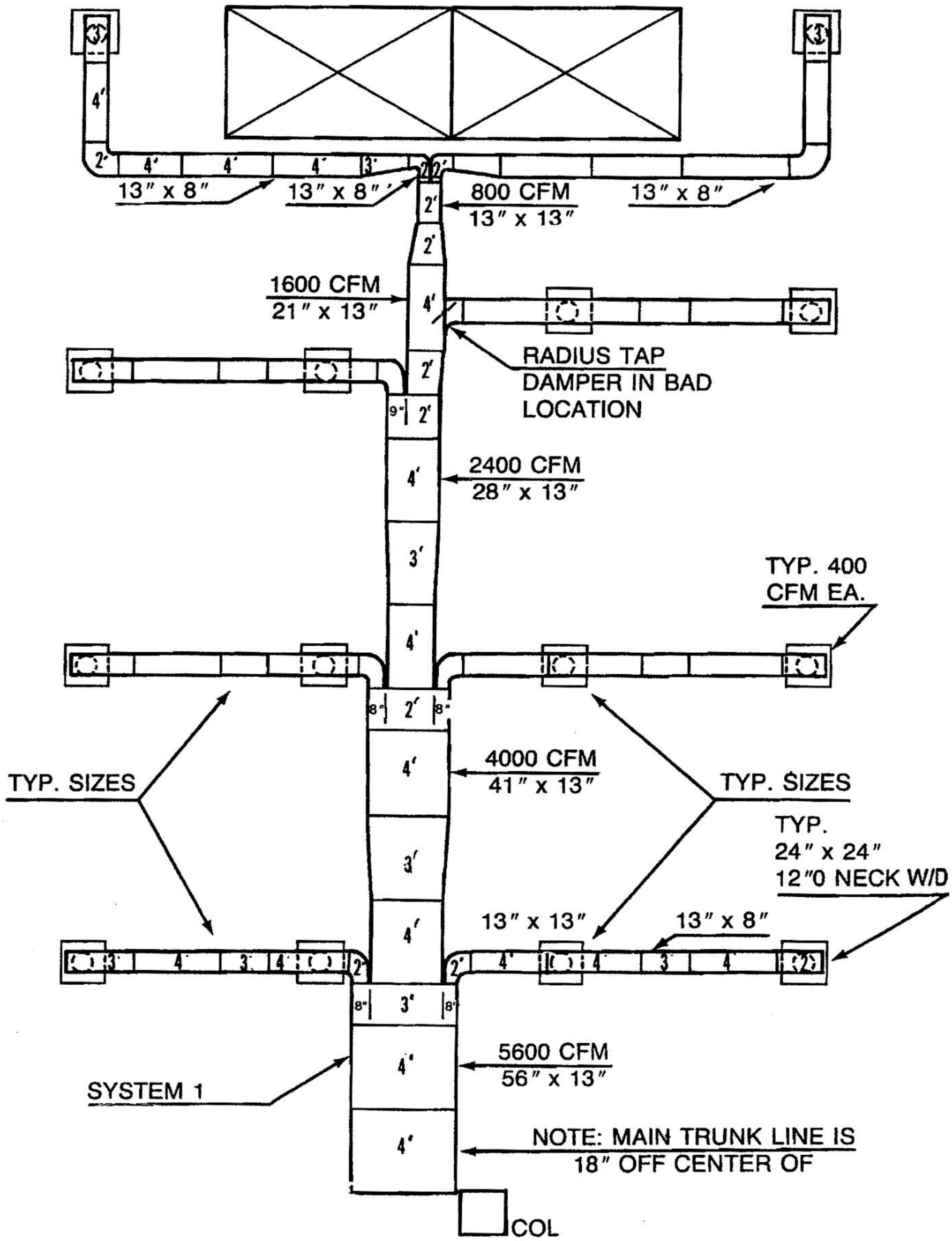


Figure 3 Typical Air duct System

I. INTRODUCTION

Due to the spreading the deadly virus, the indoor pressure must always be kept lower than atmospheric pressure. This can be accomplished by setting the airduct system pressure, static pressure portion lower than atmospheric pressure. Before we go into the pressure, negative or positive, we must first review the basic psychrometric chart as shown in the Figure 2.

The chart is based on the following equations.

$V = 1.288\ [\ V_p\,(1.2/d)\ ]^{1/2}$ (1.1)

where V is the velocity of fluid, V_P the velocity in pascal.

$SP_2\,/\,SP_1\ =\ (RPM_2\,/\ RPM_1)^2$ (1.2)

Where SP's are static pressure due to the fan operation and RPM's are the rotary fan round per minutes. Also due to the required pressure difference between indoor and outdoor for spreading the dangerous virus, we must treat the building as a pressure vessel. Or we must build a pressure vessel by itself.

The use of composite materials has been widespread over last 25 years from the aerospace vehicles to civil engineering structures, the materials possess such characteristics as high strength/density and properly controllable modulus/density ratios and they are generally composed of filaments and matrix materials. The filaments are embedded in the matrix materials to give additional stiffness and high strength. Among others, a cylindrical shell can be named as the most typical pressure vessel.

The cylindrical shell theory that we develop in this article is common to liquid oxygen storage tanks of outer space rockets, orbital shuttles as well as columns of building structures. The cylindrical coordinate system is shown in Figure 4.

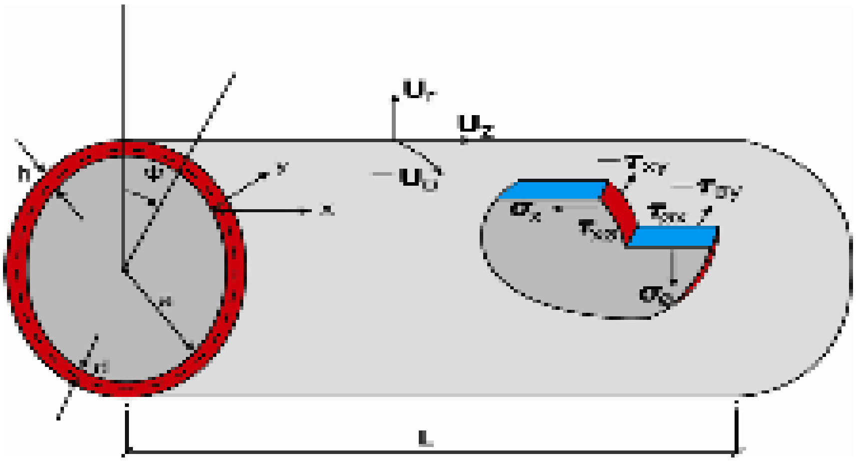


Figure 4. Dimensions, deformations and stresses of a cylindrical shell

As shown in Figure 5 the filaments can be arranged arbitrarily to make a composite structure more resistant to loadings. As the mechanical properties of composites vary depending on the direction of the fiber arrangement, it is necessary to analyze them by use of an anisotropic theory. Composite materials are also generally constructed of thin layers, which may have different thickness and different cross-ply angles. The cross-ply angle, ɣ, is the angle between major elastic axis of the material and reference axis (see Figures 5 and 6).

The variation in properties in the direction of the thickness implies non-homogeneity of the material and composite structures must thus be analyzed according to theories, which allow for non-homogeneous anisotropic material behavior. Our task is to formulate theories for a shell of composite materials, which are non-homogeneous and anisotropic, or hybrid anisotropic materials.

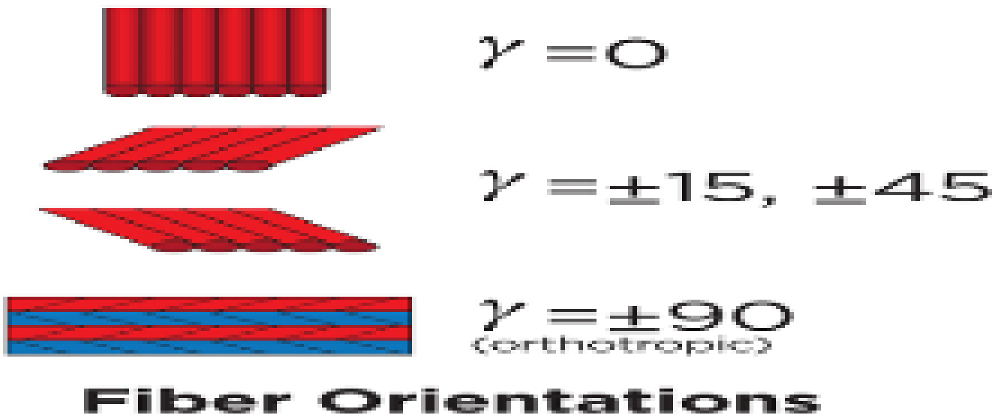


Figure 5. Fiber orientation

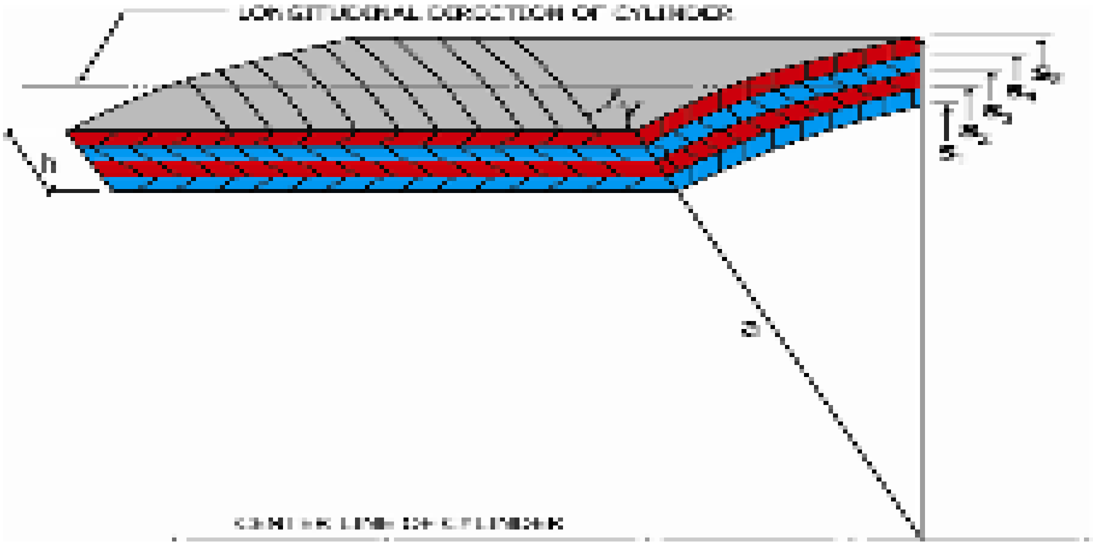


Figure 6. A laminated cylindrical shell, material orientation

According to the exact three-dimensional theory of elasticity, a shell element is considered as a volume element. All possible stresses and strains are assumed to exist and no simplifying assumptions are allowed in the formulation of the theory. We therefore allow for six stress components, six strain components and three displacements. There are thus fifteen unknowns to solve for in a three-dimensional elasticity problem. On the other hand, three equilibrium equations and six strain displacement equations can be obtained for a volume element and six generalized Hook's law equations can be used. A total of fifteen equations can thus be formulated and it is possible to set up a solution for a three-dimensional elasticity problem. It is however very complicated to obtain a unique satisfaction of the above fifteen equations and the associated boundary conditions. This led to the development of various theories for structures of engineering interest. A group of simplifying assumptions that provide a reasonable description of the behavior of thin elastic shells proposed by Love (Reference [1]) has led to the development of the classical shell theories.

Love's theory is based upon the well-known hypotheses of thin shell theory. A detailed description of classical shell theory can be found in various articles. (References [2] to [6]). The system of shell equations derived based on the assumptions is usually regarded as satisfactory for thin homogenous, isotropic shells except for problems associated with certain types of loadings and where the transverse stresses and strains may be important. For example, the transverse stresses are of importance in the so-called near edge zone of a constrained cylindrical shell under internal pressure and for a shell made of very soft material in transverse direction. This led to an examination of the classical theory of interest and numerous investigators have tried to obtain more refined theories of shells.

Another way of being free from the classical assumptions is to apply the asymptotic method to the three-dimensional elasticity equations and thus obtain so-called rational two-dimensional shell theories. Asymptotic methods have at their foundation the desire to obtain a solution that is approximately valid when a physical parameter (or a variable) of the problem which is very small (or very large). The solution is usually of a boundary or initial value problem in powers of a parameter, which either appears explicitly in the original problems or is introduced in some artificial manner. The highlights of using the asymptotic method to derive shell equations are first to comply with the three-dimensional elasticity theory and incorporates the expansion of stress and displacement components in an asymptotic series. We can then collect the first approximation by taking only the leading term of the expansions.

The derivation of the theories is accomplished by first introducing the shell coordinates and dimensions and yet unspecified characteristic length scales via changes in the independent variables. Next, the dimensionless stresses and displacements are expanded asymptotically by using the thinness of the shell as the expansion parameter. A choice of characteristic length scales is then made, and corresponding to different combination of these length scales, different sequences of systems of differential equations are obtained. Subsequent integration over the thickness and satisfaction of the boundary conditions yields the desired equations governing the formulation of the first approximation theory of non-homogeneous anisotropic cylindrical shells.

Our developed shell theory for non-homogeneous anisotropic materials is equivalent to the classical Donnell-Vlasof shell theory of homogeneous isotropic materials of a single Young's modulus of E = 29,000 ksi. (200 Gpa).

Scientists from Russian schools normally identify the Donnell's theory as Vlasof equations, we therefore name the equation as Donnell-Vlasof as described in References [2] and [6].

General Anisotropic Cylindrical Shell Theory

Based on a non-homogeneous, anisotropic volume element of a cylindrical body with longitudinal, circumferential (angular) and radial coordinates being noted as z, Θ and r, respectively, and subjected to all possible stresses and strains (Figure [5].). The cylinder occupies the space between $a \leq r \leq a + h$ and the edges are located at $z = 0$ and $z = L$. Here, a is the inner radius, h the thickness and L the length.

Assuming that the deformations are sufficiently small so that linear elasticity theory is valid, the following equili9brium and stress-displacement equations [Equations (2.1) and (2.2)] govern the problem:

$$\begin{aligned}
&(r\,\tau_{rz})_{,\,r} + \tau_{\theta z,\,\theta} + (r\,\sigma_z)_{,\,z} = 0 \\
&(r\,\tau_{r\theta})_{,\,r} + \sigma_{\theta,\,\theta} + (r\,\tau_{\theta z})_{,\,z} + \tau_{\theta z} = 0 \\
&(r\sigma_r)_{,\,r} + \tau_{r\theta,\,\theta} + (r\,\tau_{rz})_{,\,z} - \sigma_\theta = 0
\end{aligned} \qquad (2.1)$$

$$\begin{aligned}
&u_{z,z} = S_{11}\sigma_z + S_{12}\sigma_\theta + S_{13}\sigma_r + S_{14}\tau_{r\theta} + S_{15}\tau_{rz} + S_{16}\tau_{\theta z} \\
&\frac{1}{r}(u_{\theta,\theta} + u_r) = S_{12}\sigma_z + S_{22}\sigma_\theta + \cdots + S_{26}\tau_{\theta z} \\
&u_{r,r} = S_{13}\sigma_z + \cdots + S_{36}\tau_{\theta z} \\
&\frac{1}{r}u_{r,\theta} + u_{\theta,r} - \frac{1}{r}u_\theta = S_{14}\sigma_z + \cdots + S_{46}\tau_{\theta z} \\
&u_{z,r} + u_{r,z} = S_{15}\sigma_z + \cdots + S_{56}\tau_{\theta z} \\
&u_{\theta,z} + \frac{1}{r}u_{z,\theta} = S_{16}\sigma_z + \cdots + S_{66}\tau_{\theta z}
\end{aligned} \qquad (2.2)$$

In the above, u_r, u_Θ and u_z are the displacement components in the radial, circumferential and longitudinal directions, respectively, σ_r, σ_Θ, and σ_z are the normal stress components in the same directions and τ_{rz}, $\tau_{\theta z}$ and $\tau_{r\theta}$ are the shear stresses on the Θ-z face, r-z face and r-Θ face, respectively (Fig. 3). A comma indicates partial differentiation with respect to the indicated coordinates. The S_{ij}'s (i , j = 1, 2,…, 6) in (2.2) are the components of compliance matrix and represent the directional properties of the material. The compliance matrix is symmetric, $S_{ij} = S_{ji}$.

Complete anisotropy of the material is allowed for, making 21 independent material constants. We are not allowed to illiminate any of those components since the material properties depend on the manufactures set up, or, in the case of aerospace vehicles, a different gravity environment.

The components can be expressed in terms of engineering constants as follows:

$$S_{ii} = \frac{1}{E_i}, (i, j = 1, 3)$$

$$S_{ij} = \frac{-v_{ij}}{E_i}, (i = 1,2, j = 2, 3, i \neq j)$$

$$S_{44} = \frac{1}{G_{23}} \quad (2.3)$$

$$S_{55} = \frac{1}{G_{13}}$$

$$S_{66} = \frac{1}{G_{12}}$$

In (2.3), the E_i's are the Young's moduli in tension along the $i -$ direction and v_{ij} and G_{ij} are the Poisson's ratio and shear moduli in the *i-j* face, respectively. Equation (2.2) implies anisotropic property of the material only. Materials to be non-homogeneous, different properties of each layer of the shell, we will allow the material property variation in the radial direction as follows:

$$S_{ij} = S_{ij}(r) \quad (2.4)$$

Unlike the majority of theories, which identify material properties from the beginning, this theory willallow the properties to be, variable, as hybrid theory.
The principal material axes (r', θ', z') in general do not coincide with the body axes of the cylindrical shell (r, θ, z). If the material properties S'_{ij} with respect to material axes specified, then the properties with respect to the body axes are given by the anisotropic transformation equations:
The shell is free from surface traction at its inner surface while the outer surface is subjected to a uniformly distributed tensile force. The boundary condition is then:

$$\sigma_r = \tau_{r\theta} = \tau_{rz} = 0 \qquad (r = a)$$

$$\sigma_r = p(\theta, z), \ \tau_{r\theta} = \tau_{rz} = 0 \quad (r = a + h) \quad (2.5)$$

$$N_z = \int_a^{a+h} a_z \left[1 + \frac{r-a-d}{a+d}\right] dr$$

$$N_\theta = \int_a^{a+h} \sigma_\theta \, dr$$

$$N_{\theta z} = \int_a^{a+h} \tau_{\theta z} \, dr$$

$$N_{z\theta} = \int_a^{a+h} \tau_{\theta z} \left[1 + \frac{r-a-d}{a+d}\right] dr \tag{2.5}$$

$$M_\theta = \int_a^{a+h} \sigma_\theta [r-a-d] \, dr$$

$$M_z = \int_a^{a+h} \sigma_z \left[\frac{r-a-d}{a+d}\right] r \, dr$$

$$M_{\theta z} = \int_a^{a+h} \tau_{\theta z} [\mathrm{r} - \mathrm{a} - \mathrm{d}] \, dr$$

$$M_{z\theta} = \int_a^{a+h} \tau_{\theta z} \left[\frac{r-a-d}{a+d}\right] r \, dr$$

In the above, a is the inner radius of the cylindrical shell and d is the distance from the inner surface to the reference surface where the stress resultants are defined. Note that $N_{\Theta z}$ and $N_{z\Theta}$ and $M_{z\Theta}$ and $M_{\Theta z}$ respectively are different. This is due to the fact that the terms of the order of thickness over radius are not neglected compared to one in the integral expressions.

Formulation of a Boundary Layer Theory

As explained in the introduction, a theory of shells is distinguished from the exact three dimensional elasticity formulation by the fact that one of the coordinates is suppressed by the mathematical description. The procedure used here for obtaining the twodimensional thin shell equations is that of the asymptotic integration of the (2.1) and (2.2) describing the cylindrical shell. As a first step for integrating (2.1) and (2.2), we non-dimensionalize the coordinates as follows:

$$X = z/L, \ y = (r - a)/h \ , \ \varphi = \theta \ / \ \beta \tag{3.1}$$

where L and $\underline{\ell}$ $(= \ \beta a)$ are quantities which are to be determined later.

Next, the compliance matrix, the stresses and deformations are non-dimensionalized by the use of a representative stress level σ, a representative material property S and the shell radius a as follows:

$$S_{ij} = S\,\bar{S}_{ij}$$
$$\sigma_z = \sigma t_{r\theta}, \qquad \sigma_\theta = \sigma t_\theta, \qquad \sigma_r = \sigma t_r$$
$$\tau_{r\theta} = \sigma t_{r\theta}, \qquad \tau_{rz} = \sigma t_{rz}, \qquad \tau_\theta z = \sigma t_{\theta z} \tag{3.2}$$
$$u_r = \sigma a\,S v_r, \qquad u_\theta = \sigma a\,S v_\theta, \qquad u_z = \sigma a\,S v_z$$

Where the dimensionless displacements and stresses are functions of x, y and φ. These variables, together with their derivatives with respect to x, y and φ, are assumed to be of order unity. The parameters L and ℓ introduced in (3.1) are thus seen to be characteristic length scales for changes of the stresses and displacements in the axial and circumferential directions, respectively.

Consider a small parameter ϵ, $\epsilon < 1$. With respect to an arbitrary domain *D* of the cylinder, M_1 is said to be of order ϵ^n relative to a second quantity M_2.

$$M_2 \approx \varepsilon^n M_1 \tag{3.3}$$

Where n is an arbitrary between 0 and infinity

If everywhere in *D* (with the possible exception of some isolated small regions) the relation

$$\varepsilon^{n+m} \leq |M_2| / |M_1| \leq \epsilon^{n-m} \tag{3.4}$$

holds for a suitably chosen value of m, $0 < m < 1$. According to this definition, two quantities are of the same order if $n = 0$ in the above, while a quantity is of order unity when $n = 0$ and $M_1 = 1$. Substitution of the dimensionless variables defined by (3.1) and (3.2) into the elasticity equations, (2.1) and (2.2), yields the following dimensionless equations:

$$v_{r,y} = \lambda[\bar{S}_{31}t_z + \bar{S}_{32}t_\theta + \bar{S}_{33}t_r + \bar{S}_{34}t_{r\theta} + \bar{S}_{35}t_{rz} + \bar{S}_{36}t_{\theta z}]$$
$$v_{z,y} + \lambda\left(\frac{a}{L}\right) v_{r,x} = \lambda[\bar{S}_{51}t_z + \bar{S}_{52}t_\theta + \bar{S}_{53}t_r + \bar{S}_{54}t_{r\theta} + \bar{S}_{55}t_{rz} + \bar{S}_{56}t_{\theta z}]$$
$$\lambda v_{r,\emptyset} + \left(\frac{l}{a}\right)(1 + \lambda y)v_{\theta,y} - \left(\frac{l}{a}\right)\lambda v_\theta$$
$$= \lambda\left(\frac{l}{a}\right)(1 + \lambda y)[\bar{S}_{41}t_z + \bar{S}_{42}t_\theta + \bar{S}_{43}t_r + \bar{S}_{44}t_{r\theta} + \bar{S}_{45}t_{rz} + \bar{S}_{46}t_{\theta z}] \tag{3.5}$$
$$v_{z,x} = \left(\frac{L}{a}\right)[\bar{S}_{11}t_z + \bar{S}_{12}t_\theta + \bar{S}_{13}t_r + \bar{S}_{14}t_{r\theta} + \bar{S}_{15}t_{rz} + \bar{S}_{16}t_{\theta z}]$$
$$\left(\frac{a}{l}\right) v_{\theta,\emptyset} + v_r = (l + \lambda y)[\bar{S}_{21}t_z + \bar{S}_{22}t_\theta + \bar{S}_{23}t_r + \bar{S}_{24}t_{r\theta} + \bar{S}_{25}t_{rz} + \bar{S}_{26}t_{\theta z}]$$
$$\left(\frac{a}{L}\right)(1 + \lambda y)v_{\theta,x} + \left(\frac{a}{l}\right) v_{z,\emptyset} = (1 + \lambda y)[\bar{S}_{61}t_z + \bar{S}_{62}t_\theta + \bar{S}_{63}t_r + \bar{S}_{64}t_{r\theta} + \bar{S}_{65}t_{rz} + \bar{S}_{66}t_{\theta z}]$$

$$[t_{rz}(1+\lambda y)]_{,y} + \left(\frac{\lambda a}{l}\right) t_{\theta z},\emptyset + \left(\frac{\lambda a}{L}\right)(1 = \lambda y)t_{z,x} = 0$$
$$[t_{r\theta}(1+\lambda y)]_{,y} + \left(\frac{\lambda a}{l}\right) t_{\theta,\emptyset} + \lambda t_{r\theta} + \left(\frac{\lambda a}{L}\right)(1+\lambda y)t_{\theta z,x} = 0 \qquad (3.6)$$
$$[t_r(1+\lambda y)]_{,y} + \left(\frac{\lambda a}{l}\right) t_{r\theta,\emptyset} + \left(\frac{\lambda a}{L}\right)(1+\lambda y)t_{rz,x} - \lambda t_\theta = 0$$

Where λ is the thin shell parameter, defined as

$$\lambda = h/a \qquad (3.7)$$

The parameter λ is representative of the thinness of the cylindrical shell. We will consider only the case of thin shell theory as

$$\lambda \ll 1 \qquad (3.8)$$

The dimensionless coefficients S_{ij} of the compliance matrix in general are not all of same order.
We therefore assume that they can be expanded in terms of finite sum as:

$$\overline{S_{ij}}(y;\lambda) = \sum_{n=0}^{N} S_{ij}^{(n)}(y)\lambda^{\frac{n}{2}} \qquad (3.9)$$

Where $S_{ij}^{(n)}$ *(y)* is of order unity or vanish identically. Next, we assume that each displacement components, represented by the generic symbol *v* and each stress component, represented by the generic symbol *v*, and each stress components represented by the generic symbol *t*, can be expanded in terms of a power series in $\lambda^{1/2}$

$$v(y,x,\phi;\lambda) = \sum_{m=0}^{M} v^{(m)}(y,x,\phi)\lambda^{m/2}$$
$$t(y,x,\phi;\lambda) = \sum_{m=0}^{M} t^{(m)}(y,x,\phi)\lambda^{m/2} \qquad (3.10)$$

Shell Theory of Longitudinal and Circumferential Length Scales $(ah)^{1/2}$
We will now develop a theory of both characteristic length scales same as $(ah)^{1/2}$, as follows

$$L = (ah)^{\frac{1}{2}}, \quad \ell = (ah)^{\frac{1}{2}} \qquad (4.1)$$

In the above a is inner radius of cylindrical shell and h is the total thickness of the wall as shown in Figure 1 and Figure 3.
The basis of the above equations are first non-dimensionalize each and all the variables to compare the magnitudes on equal basis and empirical number, which is used in the theory of shells.
The following systems of differential equations are obtained by substituting the characteristic length scales (4.1) into (3.5) and (3.6).

$$v_{r,y} = \lambda \quad \bar{S}_{31}t_z + \bar{S}_{32}t_\theta + \quad \bar{S}_{33}t_r + \bar{S}_{34}t_{r\theta} + \bar{S}_{35}t_{rz} + \bar{S}_{36}t_{\theta z}$$

$$v_{z,y} + \lambda^{\frac{1}{2}} v_{r,x} = \lambda\bar{S}_{51}t_z + \bar{S}_{52}t_\theta + \bar{S}_{53}t_r + \bar{S}_{54}t_{r\theta} + \bar{S}_{55}t_{rz} + \bar{S}_{56}t_{\theta z}$$

$$\lambda\, v_{r,\phi} + \lambda^{\frac{1}{2}} (1 + \lambda y)\, v_{\theta,y} - \lambda^{3/2}\, v_\theta$$

$$= \lambda^{3/2}(1 + \lambda y) \quad \bar{S}_{41}t_z + \bar{S}_{42}t_\theta + \bar{S}_{43}t_r + \bar{S}_{44}t_{r\theta} + \bar{S}_{45}t_{rz} + \bar{S}_{46}t_{\theta z}$$

$$v_{z,x} = \lambda^{\frac{1}{2}}\, \bar{S}_{11}t_z + \bar{S}_{12}t_\theta + \bar{S}_{13}t_r + \bar{S}_{14}t_{r\theta} + \bar{S}_{15}t_{rz} + \bar{S}_{16}t_{\theta z}$$

$$v_{\theta,\phi} + \lambda^{\frac{1}{2}}\, v_r = \lambda^{\frac{1}{2}}(1 + \lambda y) \quad \bar{S}_{21}t_z + \bar{S}_{22}t_\theta + \bar{S}_{23}t_r + \bar{S}_{24}t_{r\theta} + \bar{S}_{25}t_{rz} + \bar{S}_{26}t_{\theta z}$$

$$(1 + \lambda y) \quad v_{\theta,x} + v_{z,\phi}$$

$$= \lambda^{\frac{1}{2}}\, (1 + \lambda y) \quad \bar{S}_{61}t_z + \bar{S}_{62}t_\theta + \bar{S}_{63}t_r + \bar{S}_{64}t_{r\theta} + \bar{S}_{65}t_{rz} + \bar{S}_{66}t_{\theta z}$$

$$v^{(0)}_{r,y} = 0 \tag{4.2}$$

$$v^{(1)}_{z,y} + v^{(0)}_{r,x} = 0$$

$$v^{(1)}_{\theta,y} + v^{(0)}_{r,\phi} = 0$$

$$v^{(1)}_{z,x} = S^{(0)}_{11}\, t^{(0)}_z + S^{(0)}_{12}\, t^{(0)}_\theta + S^{(0)}_{13}\, t^{(0)}_{\theta z}$$

$$v^{(1)}_{\theta,\phi} + v^{(0)}_r = S^{(0)}_{12}\, t^{(0)}_z + S^{(0)}_{22}\, t^{(0)}_\theta + S^{(0)}_{26}\, t^{(0)}_{\theta z}$$

$$v^{(1)}_{\theta,x} + v^{(1)}_{z,\theta} = S^{(0)}_{16}\, t^{(0)}_z + S^{(0)}_{26}\, t^{(0)}_\theta + S^{(0)}_{66}\, t^{(0)}_{\theta z}$$

$$t^{(1)}_{rz,y} + t^{(0)}_{z,x} + t^{(0)}_{\theta z,\phi} = 0$$

$$t^{(1)}_{r\theta,y} + t^{(0)}_{\theta,\phi} + t^{(0)}_{\theta z,x} = 0$$

$$t^{(2)}_{r,y} + t^{(1)}_{rz,x} + t^{(1)}_{r\theta,\phi} - t^{(0)}_\theta = 0$$

From the initial terms chosen for the first approximation system.
where v_r , v_Θ, v_z are the components of displacement at $y = 0$ surface. You will find the linear y dependence of the in-plane displacements.
We will then obtain the in-plane stress-strain relations:

$$\begin{Bmatrix} t_z^{(0)} \\ t_\theta^{(0)} \\ t_{\theta z}^{(0)} \end{Bmatrix} = [C] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} + [C] \begin{Bmatrix} K_1 \\ K_2 \\ K_{12} \end{Bmatrix} y \tag{4.3}$$

where strain components ϵ_i and curvature components k_i defined at $y = 0$, surface are given as follows:

$$\begin{aligned} \varepsilon_1 &= V^{(1)}_{z,x} & K_1 &= -V^{(0)}_{r,xx} \\ \varepsilon_2 &= V^{(0)}_{r} + V^{(1)}_{\theta,\phi} & K_2 &= -V^{(0)}_{r,\phi\phi} \\ \varepsilon_{12} &= V^{(1)}_{\theta,x} + V^{(1)}_{z,\phi} & K_{12} &= -2V^{(0)}_{r,x\phi} \end{aligned} \tag{4.4}$$

and [C] is the inverse of the first approximation compliance matrix.

$$[C] = \begin{bmatrix} S^{(0)}_{11} & S^{(0)}_{12} & S^{(0)}_{16} \\ S^{(0)}_{12} & S^{(0)}_{22} & S^{(0)}_{26} \\ S^{(0)}_{16} & S^{(0)}_{26} & S^{(0)}_{66} \end{bmatrix}^{-1} \tag{4.5}$$

The transverse stresses can now be solved by substituting the in-plane stresses obtained in (4.4) into the last equations of (4.6) and integrating with respect to *y*, we then get:

$$\begin{aligned} t^{(1)}_{rz} &= T_{rz}(x,\phi) - (A_{1j}\varepsilon_{j,x} + B_{1j}K_{j,x}) - (A_{3j}\varepsilon_{j,\phi} + B_{3j}K_{j,\phi}) \\ t^{(1)}_{r\theta} &= T_{r\theta}(x,\phi) - (A_{3j}\varepsilon_{j,x} + B_{2j}K_{j,x}) - (A_{2j}\varepsilon_{j,\phi} + B_{3j}K_{j,\phi}) \\ t^{(2)}_{r} &= T_{r}(x,\phi) - (T_{rz,x} + T_{r\theta,\phi})y + A_{2j}\varepsilon_j + B_{2j}K_j + D_{1j}\varepsilon_{j,xx} \\ &\quad + E_{1j}K_{j,xx} + 2D_{3j}\varepsilon_{j,x} + 2E_{3j}K_{j,x} + D_{2j}\varepsilon_{j,\phi} + E_{2j}K_{j,\phi} \end{aligned} \tag{4.6}$$

where T_{rz}, $T_{r\Theta}$ and T_r are the stress components at $y = 0$ and A_{ij} , B_{ij} , D_{ij} and E_{ij} are the products obtained by integration of stress strain coefficient components over the thickness coordinate. The boundary conditions are:

$$\begin{aligned} &\sigma_r = \tau_{rz} = \tau_{r\Theta} = 0 \text{ at } r = a \text{ and} \\ &\sigma_r = p\,(\Theta,z) \text{ when } r = a + h \\ &\text{also} \\ &\tau_{rz} = \tau_{r\Theta} = 0 \text{ at } r = a + h \end{aligned} \tag{4.7}$$

Where $p^* = p / (\sigma\lambda)$, to non-dimensionalize.

We now complete the second approximation theory of the asymptotic expansion and integration procedure, which can be shown as follows:

$$\begin{aligned} &v^{(2)}_{r,y} = 0 \\ &v^{(3)}_{z,y} + v^{(0)}_{r,x} = 0 \\ &v^{(2)}_{\theta,y} + v^{(0)}_{r,\phi} - v^{(0)}_{\theta} + yv^{(0)}_{\theta,y} = 0 \\ &v^{(3)}_{z,x} = S^{(0)}_{11} t^{(4)}_{z} + S^{(0)}_{12} t^{(4)}_{\theta} + S^{(2)}_{11} t^{(2)}_{z} + S^{(2)}_{12} t^{(2)}_{\theta} \\ &v^{(2)}_{\theta,\phi} + v^{(2)}_{r} = S^{(0)}_{21} t^{(2)}_{z} + S^{(0)}_{22} t^{(2)}_{\theta} \\ &v^{(2)}_{\theta,x} + v^{(3)}_{z,\phi} + yv^{(0)}_{\theta,x} = S^{(0)}_{66} t^{(3)}_{\theta z} \\ &t^{(7)}_{rz,y} + (yt^{(5)}_{rz})_{,y} + t^{(5)}_{\theta z,\phi} + yt^{(2)}_{z,x} + t^{(4)}_{z,x} = 0 \\ &t^{(6)}_{r\theta,y} + (yt^{(4)}_{r\theta})_{,y} + t^{(4)}_{\theta,\phi} + t^{(4)}_{r\theta} + t^{(3)}_{\theta z,x} = 0 \\ &t^{(6)}_{r,y} + (yt^{(4)}_{r})_{,y} + t^{(4)}_{r\theta,\phi} - t^{(4)}_{\theta} = 0 \end{aligned} \tag{8}$$

By integrating the first three equations of the above, we will obtain the following equations:

$$v_r^{(2)} = V_r^{(2)}(x,\phi) \quad , \qquad v_z^{(3)} = V_z^{(3)}(x,\phi) - V_{r,x}^{(0)} y$$
$$v_\theta^{(2)} = V_\theta^{(2)}(x,\phi) + (V_\theta^{(0)} - V_{r,\phi}^{(0)}) y \tag{9}$$

Inserting the displacements obtained in the equation (9) into the middle three equations of (8) to obtain the following equations:

$$V_{z,x}^{(3)} - V_{r,xx}^{(0)} y = S_{11}^{(0)} t_z^{(4)} + S_{12}^{(0)} t_\theta^{(4)} + S_{11}^{(2)} t_z^{(2)} + S_{12}^{(2)} t_\theta^{(2)}$$
$$S_{21}^{(0)} t_z^{(2)} + S_{22}^{(0)} t_\theta^{(2)} = V_{\theta,\phi}^{(2)} + (V_{\theta,\phi}^{(0)} - V_{r,\phi\phi}^{(0)}) y + V_r^{(2)} \tag{10}$$
$$S_{66}^{(0)} t_{\theta z}^{(3)} = V_{\theta,x}^{(2)} + V_{z,\phi}^{(3)} + 2(V_{\theta,x}^{(0)} - V_{r,x\phi}^{(0)}) y$$

By combining the equation for $V_{z,x}$ in the equation (7) and the last two equations of (9), we can obtain the in-plane stress-strain relations as follows:

$$\begin{Bmatrix} t_z^{(2)} \\ t_\theta^{(2)} \\ t_{\theta z}^{(3)} \end{Bmatrix} = [\ C\ ] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} + [\ C\ ] \begin{Bmatrix} K_1 \\ K_2 \\ K_{12} \end{Bmatrix} y \tag{11}$$

where

$$[\ C\ ] = \begin{bmatrix} S_{11}^{(0)} & S_{12}^{(0)} & 0 \\ S_{21}^{(0)} & S_{22}^{(0)} & 0 \\ 0 & 0 & S_{66}^{(0)} \end{bmatrix}^{-1}$$

$$\varepsilon_1 = V_{z,x}^{(1)}$$
$$\varepsilon_2 = V_{\theta,\phi}^{(2)} + V_r^{(2)} \tag{12}$$
$$\varepsilon_{12} = V_{\theta,x}^{(2)} + V_{z,\phi}^{(3)}$$

$$K_1 = 0$$
$$K_2 = V_{\theta,\phi}^{(0)} - V_{r,\phi\phi}^{(0)}$$
$$K_{12} = 2(V_{\theta,x}^{(0)} - V_{r,x\phi}^{(0)})$$

Substituting t_z, t_θ and $t_{\theta z}$ into the first approximation equations of (5), we will obtain the following transverse stresses:

$$t_{rz}^{(5)} = T_{rz}^{(5)}(x,\phi) - [(V^{(2)}_{\theta,x} + V^{(3)}_{z,\phi\phi})A_{33} + 2(V^{(0)}_{\theta,x\phi} - V^{(0)}_{r,x\phi\phi})B_{33}]$$
$$- [V^{(1)}_{z,xx} A_{11} + (V^{(2)}_{\theta,x\phi} + V^{(2)}_{r,x})A_{12} + (V^{(0)}_{\theta,x\phi} - V^{(0)}_{r,x\phi\phi})B_{12}]$$

$$t_{r\theta}^{(4)} = T_{r\theta}^{(4)}(x,\phi) - [V^{(1)}_{z,x\phi} A_{12} + (V^{(2)}_{\theta,\phi\phi} + V^{(2)}_{r,\phi})A_{22} + (V^{(0)}_{\theta,\phi\phi} - V^{(0)}_{r,\phi\phi\phi})B_{22}] \tag{13}$$

$$t_{r}^{(4)} = T_{r}^{(4)}(x,\phi) + [V^{(1)}_{z,x} A_{12} + (V^{(2)}_{\theta,\phi} + V^{(2)}_{r})A_{22} + (V^{(0)}_{\theta,\phi} - V^{(0)}_{r,\phi\phi})B_{12}]$$

$$[\underline{D}_{12} - (\underline{D}_{22}\underline{A}_{12}/\underline{A}_{22})]V_{z,x\phi\phi} + [\underline{A}_{11} - (\underline{A}_{12}^{2}/\underline{A}_{22})]V_{z,xxx} + [\underline{D}_{12} - (\underline{D}_{22}\underline{A}_{12}/\underline{A}_{22})]V_{z,x\phi\phi\phi}$$
$$+ [\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{\theta,\phi\phi\phi} + [\underline{B}_{12} - (\underline{A}_{12}\underline{B}_{22}/\underline{A}_{22})]V_{\theta,xx\phi} +$$
$$[\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{\theta,\phi\phi\phi\phi\phi} + [-\underline{B}_{12} + (\underline{A}_{12}\underline{B}_{22}/\underline{A}_{22})]V_{r,xx\phi} - \tag{17}$$
$$[\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{r,\phi\phi\phi\phi} - [\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{r,\phi\phi\phi\phi\phi\phi} +$$
$$(\underline{A}_{12}/\underline{A}_{22})p^{*}_{,xx} = 0$$

where

$$E_{ij} = \int_0^{y} E_{ij}\, d\rho \tag{18}$$

Finally the above equations are the governing equations of the semi-membrane theory of hybrid anisotropic materials. Note that the higher derivatives are those associated with the circumferential coordinate φ. This is due to the shorter circumferential length scale compare to the longer longitudinal length scale. The equations are further simplified if we consider the physical nature of phenomena, which are axi-symmetric load and axi-symmetric deformations and stress variations.

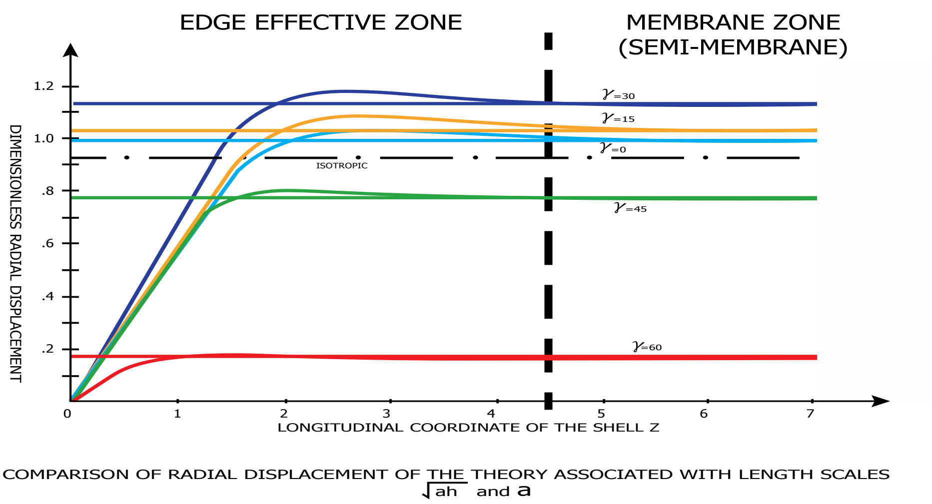


Figure 7. Radial displacement of theory $(ah)^{1/2}$ with different material angle of orientation 4 layer

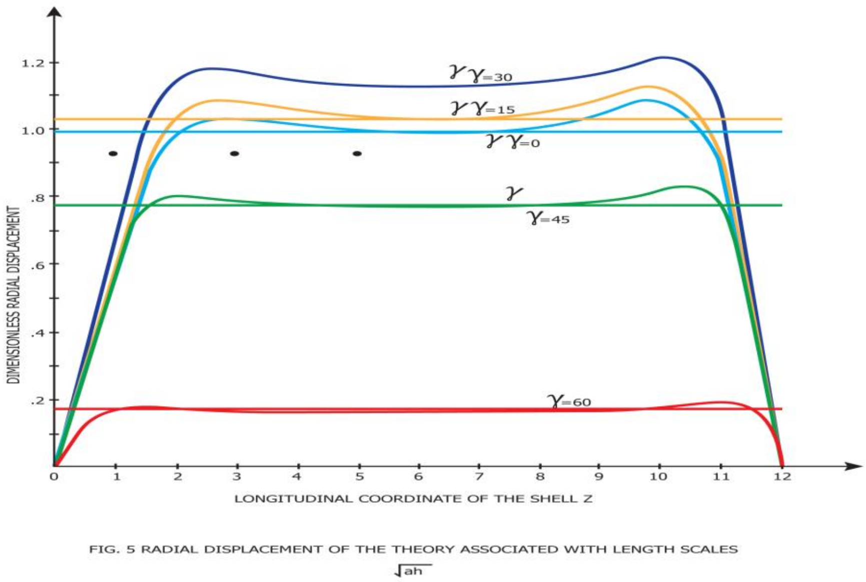


Figure 8. Radial displacement of theory $(ah)^{1/2}$ with different material angle of orientation single layer

II. CONCLUSION

A typical pressure vessel, created for keeping the deadly corona virus, was selected for the cylindrical shell of various laminated materials and it must be designed for negative pressure.

In this paper, we first discuss how the approximation shell theories are derived by use of the method of asymptotic integration of the exact three-dimensional elasticity equations for a hybrid anisotropic circular cylindrical shell.

The analysis remains valid for materials, which are non-homogeneous to the extent that their properties are allowed to vary with the thickness coordinate.

As a result of the application of this method, one can obtain shell approximate theory of various orders in a systematic manner. The first approximation theories derived in this paper represent the simplest possible shell theories for the corresponding length scales considered. Although 21 elastic coefficients are present in the original formulation of the problem, only six appear in the first approximation theories. The shell theories thus assume the existence of a plane of elastic symmetry.

It was seen that various shell theories are obtained by using different combinations of the length scales introduced in the non-dimensionalization of the coordinates and that each theory possesses unique properties such as the orders of stress magnitudes, displacement components and edge effect penetration. In this research, we considered a theory with length scales of $(ah)^{1/2}$ for both longitudinal and circumferential directions.

ACKNOWLEGEMENTS

The research was partially sponsored by Summit Partners in Menlo Park, California, USA.

REFERENCES

[1]. Love, A. E. H., A treatise on the mathematical theory of elasticity, 4th edition. New York, NY, USA: Dover Publications, 2011.
[2]. Donnel, L. H., Beams, Plates and Shells (Engineering societies monographs).: McGraw-Hill Inc.: ISBN-13: 9780070175938.
[3]. Reissner, E., "The effect of transverse shear deformation on the bending of elastic plates," J. Appl. Mech., vol 12, no A, pp. 69-77, 1945.
[4]. Johnson, M. W. and Widera, O. E., "An asymptotic dynamic theory for cylindrical shells," Studies Appl. Math., vol 48, pp. 205, 1969.
[5]. Widera, O. E., "An asymptotic theory for the motion of elastic plates," Acta mechanica, vol 9, no 54, 1970.
[6]. Vlasov, V. Z., "General theory of shells and its application in engineering", NASA TT F-99, National Tech. Information Service.
[7]. De Renzo, D.J., "Advanced Composite Materials, Products and Manufactures", Noyes Data Corporation, ISBN 0-8155-1155-8.
[8]. Heslehurst, R. B., "Design and analysis of structural joints with composite materials", DES.
[9]. Hashin, Z. and Herakovich, C. T., "Mechanics of composite materials", Pergamon Press.
[10]. Li, Changhu and Wu, Zhe, "Buckling of 120 Degree Stiffened Composite Shell", J of Composite StructuresVolume 128, Page 199, 2015.
[11]. Teeter, A. and Kolakowski, Z., "Buckling of thin walled Composite Structures with Intermediate Stiffeners", Journal of Composite Structures, Volume 69, Issue 4.
[12]. Ting, T. C. T., "Anisotropic Elasticity", Oxford Engineering Science Series 45, Oxford University Press.
[13]. Leo, D. J., "Engineering Analysis of Smart Material Systems", John Wiley and Sons, Inc.
[14]. Christensen, R. M., "Mechanics of Composite Materials", Dover Publications 2005.
[15]. Nemeth, M. P., "Conditions for Symmetries in the Buckle Patterns of Laminated Composite Plates", NASA Technical Reports Server(NTRS), 2012-217589.
[16]. Salim, H.A. and Davalos, J.F., "Torsion of Open and Closed Laminated Composite Sections", J of Composite Materials, Volume 39, Number 6, Sage Publication.
[17]. Coronaviruses, including SARS-CoV (Red Book, American Academy of Pediatrics, 2018)external icon
[18]. Severe Acute Respiratory Syndrome (SARS), CDCexternal icon
[19]. Severe Acute Respiratory Syndrome (SARS), WHOexternal icon